Enceladus’ Gravity Field
From Cassini Radio Science

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Radio Science Investigations

- Utilize the telecommunication links between spacecraft and Earth to examine changes in the phase/frequency, amplitude, and polarization of radio signals to investigate:
  - Planetary atmospheres
    - Temperature-pressure profiles
    - Composition of ionospheres
    - Winds speeds and directions
    - Scintillations & magnetic fields
    - Planetary shapes
  - Planetary interiors
    - Masses and mass distribution
    - Precision orbits
  - Planetary rings
  - Planetary surfaces
  - Solar corona and wind
  - Comet mass flux and particle distribution
  - Fundamental Physics
Gravity and Planetary Interiors

- Determine the mass and mass distribution
  - $GM$ of body or system (planet + satellites)
  - Gravity field: higher order expansion of mass distribution
- Constrain models of internal structure
  - Examples: ocean on Europa
- Improve orbits and ephemerides
- Doppler and range
  - Doppler accuracy to $\sim 0.01\text{mm/s}$ at X-band and better at Ka-band
  - Ranging accuracy to $\sim 1$ meter
Doppler Measurement of Gravity Perturbations

Line-of-sight velocity
Gravitational Potential

\[ U = \frac{GM}{r} + GM \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left( \frac{R_e}{r} \right)^{n} P_{nm} \left( \sin \phi_{lat} \right) \left[ \bar{C}_{nm} \cos(m\lambda) + \bar{S}_{nm} \sin(m\lambda) \right] \]

Normalization of Spherical Harmonic Coefficients:

\[
\begin{align*}
\left( \frac{C_{nm}}{S_{nm}} \right) & = \left[ \frac{(n-m)!(2n+1)(2-\delta_{0m})}{(n+m)!} \right]^{1/2} \left( \frac{\bar{C}_{nm}}{\bar{S}_{nm}} \right) = f_{nm} \left( \frac{\bar{C}_{nm}}{\bar{S}_{nm}} \right)
\end{align*}
\]
FLOW DIAGRAM FOR GRAVITY DATA REDUCTION

Raw Data in Cycle Counts from DSN

Data Conditioning Team
Makes Doppler at Specific Sample, Hz = 0

Time Tag on Data

Gravity User Inputs
Amount of Data
Data Weighting
Parameters to Estimate = q
Apriori Sigmas
Elevation Constraint

Gravity Inputs
Estimate of Spacecraft Position + Velocity and Model Parameters

Trajectory Program for Precision Integration of Orbit (1.5 million lines of code)

Calculation of Theoretical Data = C

Difference
O – C = Z ≠ 0

Least Square Filter
\[ \Delta q = (A^T w A)^{-1} A^T wz \]

q_{new} = q_{old} + \Delta q

\[ \frac{\partial \text{Doppler}}{\partial \text{Parameter}} = A \]

Partials

\[ \Delta q = \text{very small} \]

Trajectory Models
- Planet Ephemeris
- Earth Rotation
- Polar Motion
- Nutation
- Precession
- Tides
- Atmospheres
- Station Locations
- Spacecraft Attitude
- Relativity
- Solar Pressure
- Gravity Harmonics
- Momentum Dumps
- Maneuvers
- Planetary Constants
- GM's of Bodies
- Reference Frame J2000, EME50

Estimated Parameters
- Spacecraft State 6
- Atmosphere (Density on Each Orbit) 24
- Momentum 12
- Dump \Delta V
- Solar Pressure 3
- Data Biases 5
- Gravity Harmonics 6,000
Enceladus: facts

- Sixth largest moon of Saturn
- Orbit semi-major axis $\approx 240,000$ km
- Orbit velocity (average) $\approx 12.6$ km/s
- Orbit eccentricity $\approx 0.005$
- Orbit inclination $\approx 0.01^\circ$
- Mean radius $\approx 252$ km
- Mean density $\approx 1.61$ g/cm$^3$
- Water vapor jets from the south pole discovered in 2005
Tidal Forces: Titan
## Enceladus: gravity flybys characteristics

<table>
<thead>
<tr>
<th></th>
<th>E9</th>
<th>E12</th>
<th>E19</th>
</tr>
</thead>
<tbody>
<tr>
<td>C/A:</td>
<td>APR-28-2010 00:10:51 UTC</td>
<td>NOV-30-2010 11:53:59 UTC</td>
<td>2-MAY-2012 09:31:29 UTC</td>
</tr>
<tr>
<td>Altitude:</td>
<td>100 km</td>
<td>48 km</td>
<td>70 km</td>
</tr>
<tr>
<td>C/A latitude:</td>
<td>-89°</td>
<td>62°</td>
<td>-72°</td>
</tr>
<tr>
<td>SEP angle:</td>
<td>141°</td>
<td>54°</td>
<td>162°</td>
</tr>
<tr>
<td>Observation time:</td>
<td>-&gt; 7h continuous tracking around C/A: 2-way Doppler data only</td>
<td>-&gt; 3h continuous tracking around C/A: 3-way tracking data at C/A</td>
<td>-&gt; 3h continuous tracking around C/A: 3-way tracking data at C/A</td>
</tr>
<tr>
<td>Relative velocity:</td>
<td>6.5 km/s</td>
<td>6.3 km/s</td>
<td>7.5 km/s</td>
</tr>
</tbody>
</table>
Measurement sensitivity: gravitational accelerations

Monopole: \( \delta V^{(0)} \approx \frac{GM}{rV} \)

Degree-2: \( \delta V^{(2)} \approx \frac{GM}{rV} \left( \frac{R}{r} \right)^2 J_2 \)

Degree-3: \( \delta V^{(3)} \approx \frac{GM}{rV} \left( \frac{R}{r} \right)^3 J_3 \)

<table>
<thead>
<tr>
<th>( \delta V^{(0)} ) (km/s)</th>
<th>E9</th>
<th>E12</th>
<th>E19</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta V^{(2)} ) (km/s)</td>
<td>3.2 x 10^{-3}</td>
<td>3.9 x 10^{-3}</td>
<td>3.0 x 10^{-3}</td>
</tr>
<tr>
<td>( \delta V^{(3)} ) (km/s)</td>
<td>9.0 x 10^{-6}</td>
<td>15.0 x 10^{-6}</td>
<td>10.0 x 10^{-6}</td>
</tr>
<tr>
<td>( \delta V^{(0)} ) (km/s)</td>
<td>2.0 x 10^{-7}</td>
<td>3.0 x 10^{-7}</td>
<td>2.0 x 10^{-7}</td>
</tr>
</tbody>
</table>
Measurement sensitivity: Enceladus’ plume

The velocity variations caused by the atmospheric drag can be predicted using different models of Enceladus’ plume density profile (red boxes: Tenishev, DPS 2012; yellow boxes: Dong et al. 2011)

<table>
<thead>
<tr>
<th>Model</th>
<th>Density</th>
<th>Velocity Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>E9</td>
<td>( \rho_M = 3.78 \times 10^{-12} \text{ kg/m}^3 )</td>
<td>( \Delta V \approx \frac{1}{2} \left( \frac{\rho_M}{m_C} \right) A C_D V^2 \Delta t = 0.27 \text{ mm/s} )</td>
</tr>
<tr>
<td></td>
<td>( \Delta V \leq 0.27 \text{ mm/s} \leq \Delta V \leq 1.5 \text{ mm/s} )</td>
<td>( \Delta V = 1.03 \times 10^{-11} \text{ kg/m}^3 )</td>
</tr>
<tr>
<td>E19</td>
<td>( \rho_M = 6.46 \times 10^{-13} \text{ kg/m}^3 )</td>
<td>( \Delta V \approx \frac{1}{2} \left( \frac{\rho_M}{m_C} \right) A C_D V^2 \Delta t = 0.06 \text{ mm/s} )</td>
</tr>
<tr>
<td></td>
<td>( \Delta V \leq 0.06 \text{ mm/s} \leq \Delta V \leq 0.48 \text{ mm/s} )</td>
<td>( \rho_M = 4.85 \times 10^{-12} \text{ kg/m}^3 )</td>
</tr>
</tbody>
</table>

\( m_C \) is the mass of the particle, and \( A \) is the cross-sectional area of the particle.
Cassini Radio Science Subsystem

Two-way and three-way Doppler using X-X and X-Ka radio links. Data accuracy ~ 0.02 – 0.04 mm/s at 60 s.
Multiarc solution with Monte–Doppler residuals (X/X and X/Ka) @ 60 sec

E9 – April 2010  
RMS = 28 µm/s

E12 – November 2010  
RMS = 27 µm/s

E19 – May 2012  
RMS = 37 µm/s
Multiarc solution for global parameters

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Central value ($\times 10^6$)</th>
<th>Formal uncertainty ($\times 10^6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_2$</td>
<td>5435.2</td>
<td>34.9</td>
</tr>
<tr>
<td>$C_{21}$</td>
<td>9.2</td>
<td>11.6</td>
</tr>
<tr>
<td>$S_{21}$</td>
<td>39.8</td>
<td>22.4</td>
</tr>
<tr>
<td>$C_{22}$</td>
<td>1549.8</td>
<td>15.6</td>
</tr>
<tr>
<td>$S_{22}$</td>
<td>22.6</td>
<td>7.4</td>
</tr>
<tr>
<td>$J_3$</td>
<td>-115.3</td>
<td>22.9</td>
</tr>
<tr>
<td>$\Delta V$ (E9)</td>
<td>0.25 mm/s (92% in the direction of $-V$)</td>
<td></td>
</tr>
<tr>
<td>$\Delta V$ (E19)</td>
<td>0.26 mm/s (91% in the direction of $-V$)</td>
<td></td>
</tr>
<tr>
<td>$J_2/C_{22}$</td>
<td>3.51 $\pm$ 0.05</td>
<td></td>
</tr>
</tbody>
</table>
Estimated gravity field of Enceladus indicates:

- predominance of the quadrupole terms $J_2$ and $C_{22}$ (as expected)
- existence of a remarkable asymmetry between northern and southern hemispheres
- mild deviation of the body from hydrostatic equilibrium ($\sim 6\%$), the non-hydrostatic contributions might be small because of compensation
- small non degree-2 contributions ($J_3 \sim 0.02 J_2$)
- MOI of about 0.335-0.336 MR$^2$ compatible with a low core density of $\sim 2.4$ g/cm$^3$ and a H$_2$O mantle of density 1 g/cm$^3$ and 60 km thickness
Gravity anomalies: values and uncertainties
We inferred the presence of a liquid water reservoir at depth in the proximity of the south pole, based on a number of considerations:

- the estimated gravity anomaly is not large enough to explain the 1.2 km depression at the south pole
- this region dominance in the heat output
- the plumes activity
- the need for decoupling of the ice shell and tidal heating
Gravity measurements: interpretation (2)

Additional information concerning the ocean characteristics can be extrapolated:

- A liquid water layer (8% denser than ice) of 10 km thickness at depth would explain the observed gravity.
- The regional ocean is likely to extend out to about 50° south latitude.
- The moon is too small to have an internal energy source capable of melting the ice; tides must be the main heat source.
- The water ocean is directly in contact with the rocky core.
Conclusions

- A very fitting interpretation of Cassini gravity measurements is the presence of a regional liquid water ocean underneath the icy crust of Enceladus at the south pole.
- The water pocket functions as a tank that supplies the jets made of water-ice particles.
- A potentially habitable environment has been found in an unexpected place of the solar system, where the energy needed to produce liquid water from ice is not provided by solar radiation.
- The greater concentration of water beneath the surface at the south pole, inferred from our gravity data, fits with our understanding if how Enceladus can be active.