Destination L1: A Thematic Unit

Minimum Energy Transfer Orbits

TEACHER GUIDE – MATHEMATICS

BACKGROUND INFORMATION

In this activity, students will study the mathematics involved in having a spacecraft move from one orbit to another. They will use a mathematical formula derived from Kepler's Third Law of Motion (see "The Inclined Pendulum") to calculate the time of flight that a spacecraft would take to travel from Earth to another planet in our solar system using the minimum amount of energy.

Students begin by thinking about the type of information they would need to know in order to calculate the time of flight. From this information, they will look at the formula derived from Kepler's Third Law and make some simple calculations. Once students have calculated time of flight to one or more planets, they can use an Excel spreadsheet to investigate other transfer orbits as a technology application. This Excel spreadsheet, named after Walter Hohmann who came up with this technique in 1925, uses a surprising few equations involving simple algebra to give an insight on how spacecraft can move about our solar system. The Excel spreadsheet is a tool to make multiple calculations more efficiently. With this information, students can think about what is required to move from one body in the solar system to another.

NATIONAL SCIENCE STANDARDS ADDRESSED

(Source – *National Science Education Standards)*

Grades 9-12

Science As Inquiry
- Abilities Necessary to do scientific inquiry
- Understandings about scientific inquiry

Earth and Space Science
- Earth in the Solar System

Physical Science
- Motion and Forces

History and Nature of Science
- Historical Understanding

(View a full text of the National Science Education Standards.)

NATIONAL MATHEMATICS STANDARDS ADDRESSED

(Source – *Principles and Standards of School Mathematics)*

Grades 9-12

Algebra
- Understand patterns, relations and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.

Geometry
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Problem Solving
- Solve problems that arise in mathematics and in other contexts.

Connections

Recognize and apply mathematics in contexts outside of mathematics.

(View a full text of the Principles and Standards for School Mathematics.)

NATIONAL TECHNOLOGY STANDARDS ADDRESSED

(Source – *National Technology Education Standards)*

K-12

Technology productivity tools

Students use technology tools to enhance learning, increase productivity, and promote creativity.

Technology problem-solving and decision-making tools

Students employ technology in the development of strategies for solving problems in the real world.

9-12

Select and apply technology tools for research, information analysis, problem-solving, and decision-making in content learning.

(View a full text of the National Technology Education Standards.)

MATERIALS

For the teacher:

- Teacher Guide Supplement, "Group Summary"

For each student:

- Student Spreadsheet, "Hohmann Excel"
- Student Reporting Sheet "Minimum Transfer Orbits"
- Student Activity "Minimum Transfer Orbits"

PROCEDURE

1. Ask students to think about what they would need to know if they were going to plan a mission for a spacecraft to travel to another planet from the Earth. Allow students to work in pairs or individually to draw up a list of questions. After students have had time to think about this, ask for volunteers to respond. List the questions on the board. Some responses may include: "Where are the planets in their orbit?" "How far away are the planets from each other?" "How fast can our spacecraft go?"

2. Distribute the student activity sheet. Ask the students to view the diagram found in the background information. Ask them to describe the diagram, indicate what they know from looking at it, and what questions they may have. You may want to explain that the diagram represents two planets, their orbits (shown by the solid circles), and the path that a spacecraft might take from one planet to another and back.

3. Use the group summary technique to help students review the information contained in the background information of the student activity. Organize the students into groups of three or four. Distribute the "Minimum Transfer Orbits Student Reporting Sheet" to each student. Have students survey the student activity and identify major topics that would fit into the student reporting sheet. Students should fill in the chart individually, then check with the members of their groups for clarification or questions.

4. Draw a grid on the board and have volunteers from each group contribute to the class chart. Record the information in complete sentences. Conduct a class discussion during this whole class review

Alternate Strategy Tip

Challenge students to use the formulas to create a spreadsheet to find the travel time of a spacecraft from one planet to another using Hohmann minimum energy transfer orbits.

Use the technology application below as an extension of the information contained in the student activity. Emphasize the concepts that are most important and ensure that they are written clearly. Use the Teacher Guide Supplement to help guide this discussion.

5. Ask students to view the “Table of Periods for Orbits of Some Bodies in Space” found in the student activity. Ask questions similar to the following:
   - How many kilometers are in one astronomical unit (AU)? (149,632,000)
   - Based on the information on the chart, what is the definition of one astronomical unit (AU)? (Students may suggest that one AU is the distance the Earth is from the sun.)
   - How does the distance between Mars and the sun compare with the distance between the Earth and the sun? (The distance between the sun and Mars is about 1.5 times greater than the distance between the
   - Earth and the sun.)
   - Why is the astronomical unit a convenient unit when measuring the distance of objects in the solar system? (Answers will vary. Students may suggest that AU is a larger distance than kilometer making large distances in the solar system easier to comprehend than miles or kilometers. Others may suggest that using the Earth–sun distance as a reference is a convenient reference distance.)
   - William Herschel discovered Uranus in 1781. What did this discovery do to the size of the known solar system? (Students may suggest that it doubled the size. Uranus is sometimes called “the solar system
   - doubler.”)

6. Instruct students to calculate the Hohmann minimum energy transfer orbit time of flight for a trip from the Earth to Venus. They may use the example to Mars as a guide. Students should find that the time should be close to 0.4 years. You may ask the students to complete similar calculations to the other planets. The following technology application can be used if you would like to engage students to use an Excel spreadsheet.

Alternate Strategy Tip

1. Instruct students to open the Student Spreadsheet, “Hohmann Excel” and follow the procedure on the student activity. After completing the activity, ask them to answer the questions that follow on the “Minimum Transfer Orbits Student Reporting Sheet.” Suggested answers to the questions are included below.

- What is the orbital velocity of the Earth? (29.72 km/sec)
- Which planet has the fastest orbital velocity? What is its velocity? (Mercury 47.77 km/sec)
- If we are traveling to Mars, what would the velocity of our spacecraft be as it moves along the elliptical transfer orbit when it meets the circular orbit of Mars? (column N) (21.43 km/sec)
- Based on the information in column J, would our spacecraft have to increase or decrease velocity to enter into orbit with Mars? (increase velocity to 24.08 km/sec)
- How many days would this flight take? (259.35 days)

Change the spreadsheet, so that the transfer orbits are from Mars to the other planets, by placing the distance in cell C5 into cell D2, then answer the following questions:

- How many years would it take to get to Jupiter from Mars? (3.09 years)
- How fast do we need to get the spacecraft to go in the transfer orbit to get to Jupiter? (29.95 km/sec)
- How much faster is this than Mars’ orbital velocity? (5.87 km/sec)

2. Invite students to complete one or more of the extension activities that follow the questions.

3. Ask students to read “A Final Important Note.” Ask them to summarize this information in the Group Summary Sheet. Instruct students to write a paragraph about the benefits and limitations of this Excel spreadsheet as one plans a trip to the planets.

TEACHER RESOURCES

Heidema, Clare, and Barton, Mary Lee. (2000). Teaching Reading in Mathematics: A Supplement to Teaching Reading in the Content Areas: If Not Me Then Who? Teacher's Manual. Mid-continent Research for Education and Learning.

URL

http://rigel.neep.wisc.edu/~jfs/neep533.lecture9.trajectories.99.html
Spacecraft Trajectories