Impact risk to Martian satellites due to dust in the coma of C/2013 A1 (Siding Spring)

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Comet C/2013 A1 was discovered earlier this year by Australian National University (ANU) astronomer Robert H. McNaught at Siding Spring observatory. After additional observations and the identification of prediscovery images from the Catalina Sky Survey, the comet was determined to be on a nearly parabolic orbit (McNaught et al. 2013). As of April 18, 2013, the JPL Small-Body Database[1] classifies the orbit as hyperbolic, with an eccentricity of 1.0004 and a predicted pericenter distance of 1.4 AU.

C/2013 A1 is notable in that it is projected to have a close encounter with Mars on October 19, 2014 at 18:51± 0:47 (UT). A collision between the comet and the planet has been ruled out and the close approach distance is currently estimated at 113,000 km, with a minimum of 8900 km and a maximum of 297,000 km. Because cometary comae are hundreds of thousands of kilometers in radius, Mars will almost certainly pass through the coma and tail generated by C/2013 A1.

Mars has three operational manmade satellites in orbit: NASA's Mars Reconnaissance Orbiter, NASA's Mars Odyssey, and ESA's Mars Express. One additional satellite, NASA's MAVEN (Mars Atmosphere and Volatile EvolutioN), is scheduled to arrive at Mars roughly one month prior to the close encounter with C/2013 A1.[2] Characterizing the dust environment is key to assessing the risk posed to these satellites by dust in the comet's coma.

In this analysis, we estimate the total fluence of dust particles 100 microns or larger along Mars's projected trajectory; this fluence can be combined with satellite parameters to obtain a risk of impact. Our algorithm is as follows:

1. Extrapolate the total brightness of the coma at the time of close encounter from current observations.
2. Translate this brightness into a total dust surface area via an assumed dust albedo.
3. Translate dust surface area into dust number density via an assumed particle size distribution.
4. Integrate along Mars's trajectory through the coma to obtain the total fluence, assuming a spherically symmetric dust distribution.

[1] `http://ssd.jpl.nasa.gov/?horizons`

[2] `http://www.space.com/18977-nasa-maven-mars-mission-preparation.html`

The above steps allow us to estimate the fluence, to within an order of magnitude, without simulating the coma dynamics. This simplified method is desirable because it illustrates the dependence of the total fluence on observables, and thus enables quick recalculation of expected fluence as new observations are made. However, we do supplement this analytical approach with a set of dynamical simulations performed by Paul Wiegert of the University of Western Ontario. The total number of particles in these simulations is essentially a free parameter and thus does not provide a check on the overall fluence. Rather, we include these simulations to investigate the degree of coma asymmetry and the size-dependence of the dust's spatial distribution.

In order to model the coma dust environment, we rely on several observational studies, particularly data taken in the coma of 1P/Halley by the Giotto spacecraft's Dust Impact Detection System (DIDSY), detailed in McDonnell et al. (1986) and further analyzed by Fulle et al. (2000), among others. We acknowledge that 1P/Halley is a short-period comet with a perihelion distance half that of C/2013 A1 (Siding Spring); therefore, we limit our reliance on 1P/Halley to the general form of the dust size distribution and spatial distribution. We then supplement this with quantizations of dust physical properties (for instance, density) from other studies. Finally, we perform a self-consistency check by applying our model to the coma of 1P/Halley and comparing the results to the fluence on spacecraft Giotto.

1. Cometary magnitudes

The apparent magnitude of a comet, m, as well as that of other small bodies, follows the relation

$$m = H + 5 \log \Delta + 2.5 n \log r \,, \tag{1}$$

where H is the absolute cometary magnitude, Δ is the distance in AU between the comet and the observer, and r is the distance, also in AU, between the sun and the comet. The quantity n is a separate observable that describes the dependence of the objects brightness on heliocentric distance. Below, we derive this equation and relate these quantities to physical properties of the comet.

Apparent magnitudes in general follow the relation

$$m_c - m_{\odot,1\mathrm{AU}} = -2.5 \log \frac{F_{c,r,\Delta}}{F_{\odot,1\mathrm{AU}}} \,. \tag{2}$$

This relation works for any two bodies, but we've chosen to compare our comet to the sun; $F_{c,r,\Delta}$ represents the light flux of the comet at the observer's location, while $F_{\odot,1\mathrm{AU}}$ is the light flux of the sun at 1 AU. At 1 AU, the sun has an apparent magnitude $m_{\odot,1\mathrm{AU}} = -26.74$.

The reflectivity of asteroids and comets is measured in terms of the geometric albedo, or reflectance at zero phase angle. This reflectance is expressed relative to the geometric albedo of a

Lambertian disk – the intensity of light reflected by a perfectly diffusive Lambertian disk at zero phase angle is four times that of an isotropic reflector such as a metallic sphere (van de Hulst 1981; Barbieri 2007). Hence, the flux at zero phase angle for such an object is I_{tot}/π rather than $I_{tot}/4\pi$.

By assuming that a comet's brightness is due to reflected solar light, we can express the flux as follows:

$$F_{c,r,\Delta} = \frac{F_{\odot,r}}{\pi\Delta^2} \cdot aA(r) = \frac{F_{\odot,1\mathrm{AU}}}{\pi\Delta^2(r/1\mathrm{AU})^2} \cdot aA(r) \tag{3}$$

where a is the albedo and $A(r)$ is the illuminated area, or cross-section. A comet produces a coma in response to solar irradiation; as it nears the sun, volatiles sublimate and accelerate dust particles away from the nucleus. As a result, the total dust cross section is a function of heliocentric distance. If we assume $A(r) = A_0(r/1\mathrm{AU})^{-\beta}$, where A_0 and β are constants, the above equation becomes

$$F_{c,r,\Delta} = \frac{F_{\odot,1\mathrm{AU}}}{\pi\Delta^2} \cdot aA_0(r/1\mathrm{AU})^{-(2+\beta)}\,, \tag{4}$$

Insertion of the above relation into Equation 2 results in

$$m_c - m_{\odot,1\mathrm{AU}} = -2.5\log\left(\frac{aA_0}{\pi\Delta^2} \cdot (r/1\mathrm{AU})^{-(2+\beta)}\right), \tag{5}$$

or, with some rearranging,

$$m_c = m_{\odot,1\mathrm{AU}} + 5\log\left(\frac{\Delta}{1\mathrm{AU}}\right) + 2.5(2+\beta)\log\left(\frac{r}{1\mathrm{AU}}\right) - 2.5\log\left(\frac{a}{\pi}\cdot A_0\right). \tag{6}$$

We thus recover the form of Equation 1, where

$$H = m_{\odot,1\mathrm{AU}} - 2.5\log\left(\frac{a}{\pi}\cdot A_0\right) \tag{7}$$

$$n = 2+\beta \tag{8}$$

Typically, $n \sim 4$ for comets (examples available from JPL HORIZONS); this translates to $\beta \sim 2$, or that the illuminated area of a comet follows an inverse-square law. Stated differently, the production of coma material is, to first order, proportional to the intensity of solar radiation incident on the comet. We can also use this equation for asteroids. In the case of asteroids, there is no coma, and hence the surface area is independent of heliocentric distance (i.e., $\beta = 0$; see Appendix A for details).

2. Dust Abundance in Cometary Comae

We rearrange Equation 7 and replace H with M1, which is more commonly used to denote total cometary magnitudes, in order to determine the total cross section of coma particles:

$$A_0 = g\frac{\pi}{a}10^{-0.4(M1-m_{\odot,1\mathrm{AU}})}\,. \tag{9}$$

Note that we have introduced a factor, g, to represent the fractional dust contribution to the comet's total brightness. We set this value to be unity for most of this analysis but retain g in our expressions. Fulle et al. (2000) found that for certain values of density and albedo, the dust could account for the total brightness of the coma ($g \sim 1$), but for large dust material density ($\rho = 1$ g/cc), $g \sim 0.5$.

At a given heliocentric distance, the dust total cross section is then

$$A(r) = g\frac{\pi}{a}10^{-0.4(M1-m_{\odot,1\mathrm{AU}})}r^{-\beta}\,. \tag{10}$$

2.1. Dust size distribution

To extract additional information about coma dust, we must make some assumptions about the dust distribution. We adopt a simple power law (with exponent k) for the particle size probability distribution function:

$$f(s) = Cs^{-k}\,. \tag{11}$$

where s represents dust particle radius and C is a multiplicative factor. Then, the total mass, M_c, total cross-sectional area, A_c, and total number, N_c, of dust particles in the coma can be expressed in terms of $f(s)$:

$$M_d = \rho\frac{4}{3}\pi\int_{s_{\min}}^{s_{\max}} s^3 f(s)\mathrm{d}s \tag{12}$$

$$A_d = \pi\int_{s_{\min}}^{s_{\max}} s^2 f(s)\mathrm{d}s \tag{13}$$

$$N_d = \int_{s_{\min}}^{s_{\max}} f(s)\mathrm{d}s \tag{14}$$

Next, we equate Equations 10 and 13 to determine the constant C at the point of pericenter passage, at which $r = q$:

$$C = \frac{g}{aq^{\beta}}10^{-0.4(\mathrm{M1}-m_{\odot,1\mathrm{AU}})}\frac{3-k}{s_{\max}^{3-k}-s_{\min}^{3-k}} \tag{15}$$

2.2. Fluence of particles larger than 100 microns

In order to calculate the number density, ν, of dust particles, we assume a constant outward flux of dust particles. Thus,

$$\nu(r) = Dr^{-2}\,, \tag{16}$$

where D is another multiplicative factor. One expects an inverse-square dependence on radius if the comet steadily emits particles with a constant ejection velocity distribution; in this case, the coma size is determined by ejection speed and the duration of the comet's period of activity. In actuality, ejection velocity will be a function of heliocentric distance (Whipple 1951; Jones 1995; Crifo & Rodionov 1997) and ejection is by no means steady and isotropic. However, $\nu \propto r^{-2}$ is a reasonable first order approximation (Fulle et al. 2000).

We can determine the constant D by equating the particle size distribution integral with the particle spatial distribution integral:

$$N_d = \int_{r_{\rm min}}^{r_{\rm max}} \nu(r) \cdot 4\pi r^2 {\rm d}r = 4\pi D(r_{\rm max} - r_{\rm min}) \tag{17}$$

$$= \int_{s_{\rm min}}^{s_{\rm max}} f(s){\rm d}s = C \cdot \frac{s_{\rm max}^{1-k} - s_{\rm min}^{1-k}}{1-k} \tag{18}$$

where $r_{\rm max} = r_c$ describes the radius of the coma, and $r_{\rm min} \approx 0$ is the radius of the nucleus. Therefore,

$$D = C\frac{s_{\rm max}^{1-k} - s_{\rm min}^{1-k}}{4\pi r_c(1-k)} \tag{19}$$

Finally, we calculate the fluence of particles along a straight trajectory through the coma. We express this fluence, σ, in terms of impact parameter, or distance of closest approach, b:

$$\sigma(b) = \int_{x_1}^{x_2} \nu(\vec{x}){\rm d}x = 2\int_0^{x_2} \nu(\vec{x})dx \tag{20}$$

$$= 2D\int_0^{x_2} \frac{{\rm d}x}{x^2+b^2} = 2\frac{D}{b}\tan^{-1}\left(\frac{x}{b}\right)\Big|_0^{x_2} \tag{21}$$

$$\sigma(b) = \frac{2D}{b}\cos^{-1}\left(\frac{b}{r_c}\right) . \tag{22}$$

3. Application to C/2013 A1 (Siding Spring)

In order to apply our approach to estimate the fluence of particles, we must adopt a number of physical parameters for C/2013 A1 (Siding Spring), many of which cannot be measured without a spacecraft mission (parameters are summarized in Table 1). Therefore, we determine the fluence in terms of its dependencies on various physical parameters.

We have adopted the dust size distribution model of Fulle et al. (2000), which was developed using impact data obtained from the flight of Giotto through the coma of 1P/Halley. Giotto probed dust masses ranging from 7.71×10^{-9} to 31.0 g (Fulle et al. 2000); we are interested in dust particles for which $m > m_* = 4.19 \times 10^{-6}$ g or larger (i.e., particles which are larger than 100 μm when $\rho = 1$ g/cc). Note that dust size is a function of density as well as mass. Furthermore, while the entire mass range contributes to the brightness of the coma, we are interested in the flux of particles large enough to damage a spacecraft. Thus, we replace the lower size limit in Equation 19 with m_*. With these substitutions, and combining Equations 15, 19, and 22, we obtain the relation:

$$\sigma_* = \frac{gq^{-\beta}}{a}\left(\frac{2}{\pi}\right)^{\frac{1}{3}}\left(\frac{\rho}{3}\right)^{\frac{2}{3}} 10^{-0.4(M1-m_{\odot,1\mathrm{AU}})}\left(\frac{3-k}{1-k}\right)\left(\frac{m_{\max}^{(1-k)/3}-m_*^{(1-k)/3}}{m_{\max}^{(3-k)/3}-m_{\min}^{(3-k)/3}}\right)\cdot\frac{\cos^{-1}(b/r_c)}{b\ r_c} \quad (23)$$

From the above equation we can see that the fluence of large particles has a simple dependence on most of our parameters; for instance, $\sigma \propto a^{-1}$.

The dependence on k is more complex. For large negative values of k, large particles dominate the dust contribution to coma brightness, and $\sigma \propto m_{\max}^{-2}$. For large positive values of k, we instead see $\sigma \propto (m_*/m_{\min})^{-k}$. We plot the behavior of σ relative to k for intermediate values in Figure 1; we note that the fluence has a maximum at $k = 3.27$, at which σ is enhanced by a factor of 2.5 relative to its value at our nominal choice of $k = 2.6$.

3.1. Impact risk to Martian satellites

Using the available information for comet C/2013 A1 and supplementing where necessary with typical physical cometary properties (Table 1), we have determined that the fluence of 100μm or larger particles near Mars during the comet's close encounter will be roughly 0.2 particles per square meter (Figure 2).

However, as noted in our discussion of Equation 23, this fluence may be enhanced if parameters differ from those in Table 1. This enhancement is bounded in one case – a different value of k can, at most, enlarge σ by a factor of 2.5 – but not in others. To illustrate these effects, we express the fluence at $b_{113} = 113,000$ km in terms of these physical parameters:

$$\sigma_*(b_{113}) \lesssim 0.2\ \mathrm{m}^{-2} f_k \cdot g \cdot (1.4)^{-(\beta-2.4)}\left(\frac{a}{0.04}\right)^{-1}\left(\frac{\rho}{0.1\ \mathrm{g/cc}}\right)^{\frac{2}{3}} 10^{-0.4(\mathrm{M1}-5.2)}\,, \quad (24)$$

where $f_k = \sigma(k)/\sigma(k = 2.6) \leq 2.5$.

Thus, the impact probability is roughly 20% per square meter of spacecraft. We next discuss the possible variation in this quantity due to parameters in Equation 24.

3.2. Dependence on dust properties

Although a "typical" cometary albedo is about 0.04 (see, for example, Fulle et al. 2000), much smaller values have been measured. For instance, Lacerda & Jewitt (2012) determined the albedo of dust in the coma of comet 17/P Holmes to be 0.006, and Fernández (2000) measured the albedo of fellow Oort cloud comet Hale-Bopp at 0.01. Lower albedos translate to higher expected fluences.

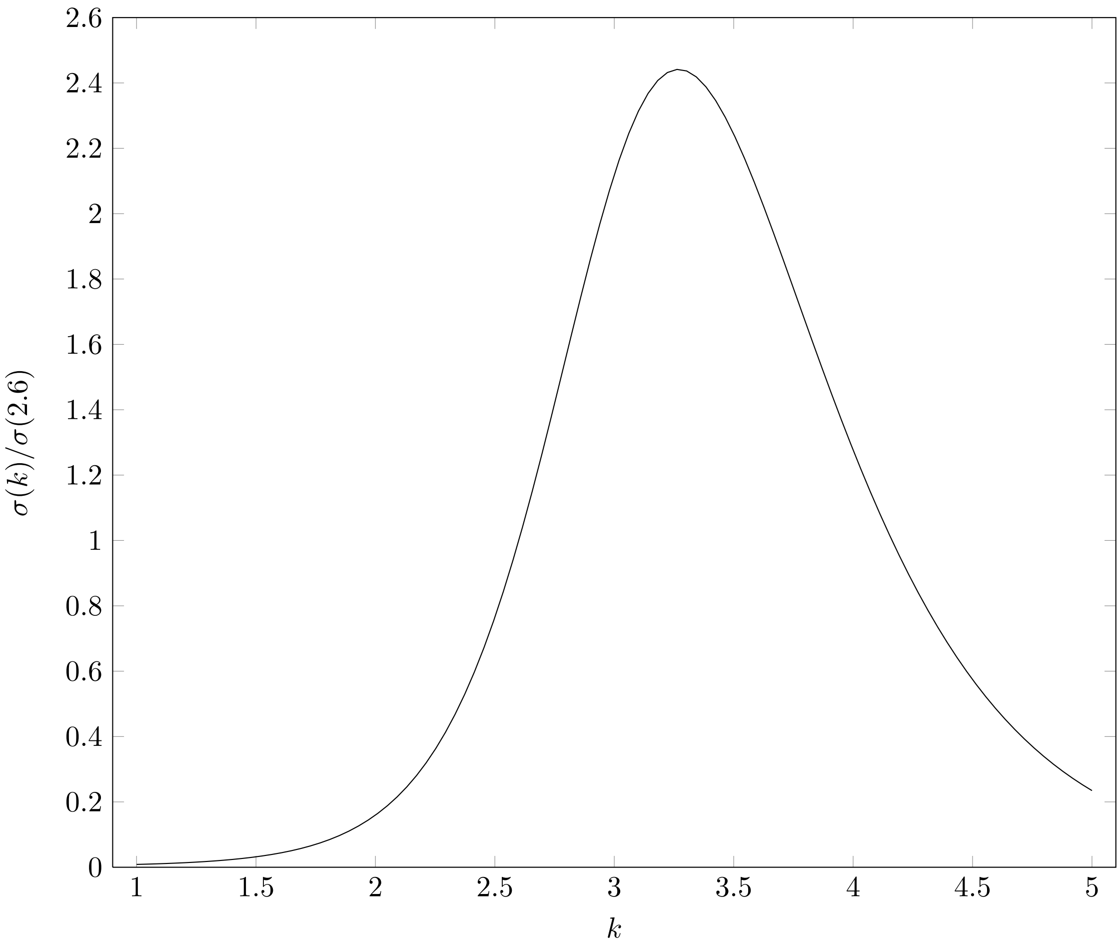

Fig. 1.— Fluence as a function of k relative to the nominal value at $k = 2.6$, where k is the exponent of the dust size probability distribution. Note that variation in k can, at most, enhance the fluence by a factor of 2.5 relative to that obtained using our default value.

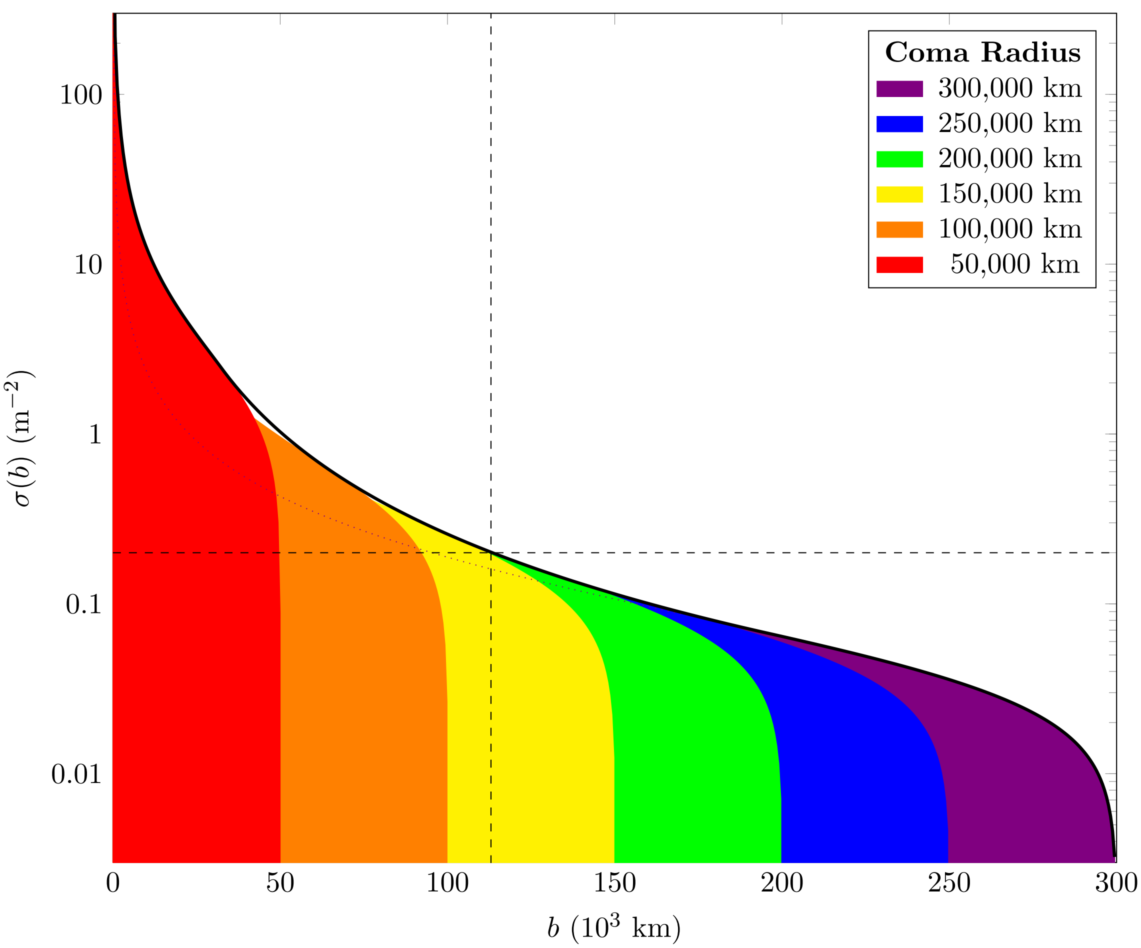

Fig. 2.— Fluence as a function of distance of closest approach, b, for a suite of coma radii. Fluences for smaller coma radii are overlaid onto fluences for larger coma radii; i.e., red ($r_c = 5 \times 10^4$ km) on top of orange ($r_c = 10^5$ km). The solid black curve marks the maximum fluence at every value of b, obtained by varying r_c. The dotted purple curve depicts the fluence for our largest choice of coma radius, 300,000 km. The vertical dashed line marks the expected distance of closest approach between C/2013 A1 (Siding Spring) and Mars, which JPL lists as 113,000 km as of April 11, 2013. The horizontal dashed line marks the maximum possible fluence at this distance ($\sigma = 0.2$) given our choices of physical parameters.

If, however, multiple reflections occur, as suggested by Larson & A'Hearn (1984), the effective dust albedo could be higher, resulting in a low fluence despite a low true albedo.

If the dust particles are simultaneously denser (say, $\rho = 1$ g/cc) and less reflective ($a = 0.01$), the fluence could be as large as 3.3 particles per m^2. However, models predict a correlation between albedo and density, with lower albedos occurring for more porous, less dense particles (Hage & Greenberg 1990). If, as assumed by Fulle et al. (2000), ρ/a is constant at 2.5 g/cc, fluence will have a shallower dependence on density ($\sigma_* \propto \rho^{-1/3}$). In this case, a larger density actually produces a lower fluence. According to Fulle et al. (2000), at some point this constant ratio does not hold; they assert that a value $\rho = 1$ g/cc is not accompanied by a similarly large value of albedo and the dust cannot completely account for the brightness of the coma. We note that choosing $\rho = 1$ g/cc, $a = 0.04$, and $g = 0.5$ produces a 2.3-fold increase in fluence.

Kelley et al. (2013) also argue in favor of low density: *Deep Impact* detected significant numbers of large ($\gtrsim 1$ cm) particles in the coma of 103P/Hartley 2, and the authors noted significant asymmetry in the spatial distribution of these particles. Kelley et al. (2013) argue that radiative forces are not capable of redistributing such large particles unless said particles have low density ($\rho \lesssim 0.1$ g/cc). Additionally, the same study argues in favor of a steep particle size distribution ($4.7 < k < 6.6$); according to Figure 1, this would translate to a lower dust fluence in our model.

3.3. Dependence on cometary properties

The uncertainty in fluence due to dust properties pales in comparison to that resulting from cometary magnitude and orbit. Figure 2 demonstrates that if the comet passes within a few thousand kilometers of Mars and its satellites, the result will be hundreds of impacts per square meter.

We would also like to point out that C/2013 A1 (Siding Spring) is currently at a heliocentric distance of 6.45 AU (according to JPL HORIZONS as of April 4, 2013). Thus, the current estimate for absolute cometary total magnitude, M1, is based on observations made solely while the comet lies outside the so-called "snow-line." JPL HORIZONS cites an uncertainty of M1 $= 5.2 \pm 0.4$; using Equation 24, we determine that if the magnitude is at the brighter end of this range (i.e., M1 $= 4.8$), the dust abundance will increase by 50%. Similarly, if β is closer to Halley's value of 1.2 than the current value of 2.4, this also translates to a 50% increase in dust.

3.4. Comparison with simulations

Our model assumes spherical symmetry, yet observations and simulations demonstrate that cometary comae are asymmetric (Schwarz et al. 1997; de Val-Borro et al. 2012; Vincent et al. 2013; Kelley et al. 2013). The true shape of comae is likely to be better determined by numerical simulations, to which we can apply a normalization factor:

$$\sigma_{\rm corr} = \frac{N}{N_{\rm sim}} = \frac{gq^{-\beta}}{aN_{\rm sim}} 10^{-0.4({\rm M1}-m_{\odot,1{\rm AU}})} \left(\frac{3-k}{1-k}\right) \left(\frac{s_2^{1-k} - s_1^{1-k}}{s_{\rm max}^{3-k} - s_{\rm min}^{3-k}}\right) \quad (25)$$

where $N_{\rm sim}$ represents the total number of particles simulated between sizes s_1 and s_2.

As volatiles in the comet nucleus sublimate, the resulting gas pressure drives solid particles outward. As time passes, the shape of this cloud of gas and dust deviate from a sphere due to the influence of gravitational forces and solar radiation. As a comet nears the sun and dynamical timescales shorten, this asymmetry can take the form of an extended cometary tail. However, the dust tail is not a distinct dynamical feature; there is a continuum between coma and tail in which the degree of asymmetry depends on factors such as heliocentric distance and particle size. We take the entire dust component of the coma and tail continuum into account by simulating the ejection and evolution of dust particles from comet C/2013 A1.

The following numerical simulations of particle ejection from the nucleus of C/2013 A1 and their subsequent dynamical evolution were performed by Dr. Paul Wiegert (University of Western Ontario). These simulations use the ejection velocity model of Jones & Brown (1996), in which

$$v_{ej} = 41.7{\rm m/s}\ R_N^{1/2} m^{-1/6} \rho^{-1/3} r^{-1.038}\,, \quad (26)$$

where R_N is the radius of the nucleus in km, m is the mass of the ejected particle in kg, ρ is the bulk density of the dust in kg/m^3, and r is the heliocentric distance of the comet in AU. Note that ejection velocity is a function of particle size; unlike in our spherical model, we expect the spatial distribution to vary as a function of particle size.

Figure 3 compares our spherical model with simulations; these plots represent fluence in the plane containing Mars (see Appendix B for exact location of the subradiant) and perpendicular to C/2013 A1's velocity relative to Mars. We compare results for two different mass regimes: $m > 4.19 \times 10^{-6}$ g (which corresponds to a grain radius of 100 μm at a density of 1 g/cc) and $m > 4.19 \times 10^{-3}$ g (which corresponds to 1000 μm, or one millimeter). We see that, as expected, small grains are more diffusely distributed than large grains, reducing the fluence relative to our spherical model within the coma. Results shown in Figure 3 are calculated using a dust bulk density of 1 g/cc and a dust contribution factor of 0.5.

We can quantify the dependence of coma size on particle size by combining our size-dependent ejection velocity with the length of the period of activity to determine coma radius. Our simulations begin when C/2013 A1 is at 10 AU, which occurs around January 1, 2012, making the total duration 1461 days. Our calculated coma radii for each size bin correspond closely with the distribution seen in simulation results (Figure 5); values are given in Table 2.

We repeat our analysis using our default parameters: $\rho = 0.1$ g/cc and $g = 1$. We adjust the coma radius accordingly, multiplying by $\rho^{-1/3} = 2.15$. The result (Figure 4) is a lower total number of massive particles spread out over a larger area, further lowering the fluence at Mars and the impact risk to Martian spacecraft.

Parameter	Symbol	Value	Reference
Total magnitude	M1	5.2	JPL
Heliocentric distance	$r = q$	1.4 AU	JPL
Radial dependence exponent	$\beta = n - 2$	2.4	JPL
Approach distance	b	113,000 km	JPL
Albedo	a	0.04	Fulle et al. (2000)
Density	ρ	0.1 g/cc	Fulle et al. (2000)
Dust contribution fraction	g	1	Fulle et al. (2000), Kelley et al. (2013)
Size distribution exponent	k	2.6	Fulle et al. (2000)
Minimum dust mass	$m_{\rm min}$	7.71×10^{-9} g	Fulle et al. (2000)
Maximum dust mass	$m_{\rm max}$	31.0 g	Fulle et al. (2000)
Coma radius	r_c	variable	

Table 1: Key orbital and physical parameters for comet C/2013 A1 (Siding Spring), our default values, and sources.

$s_{\rm min}$	v_{ej}	r_c	$r_{c,\rm fit}$
10 μm	30 m/s	3.8×10^6 km	5.68×10^6 km
100 μm	9.5 m/s	1.2×10^6 km	1.28×10^6 km
1000 μm	3.0 m/s	378×10^3 km	360×10^3 km
1 cm	0.95 m/s	120×10^3 km	120×10^3 km

Table 2: Coma radii as a function of particle size: r_c is the radius as calculated using Equation 26, $r_{c,\rm fit}$ is the radius that provides the best fit to simulation data as shown in Figure 5.

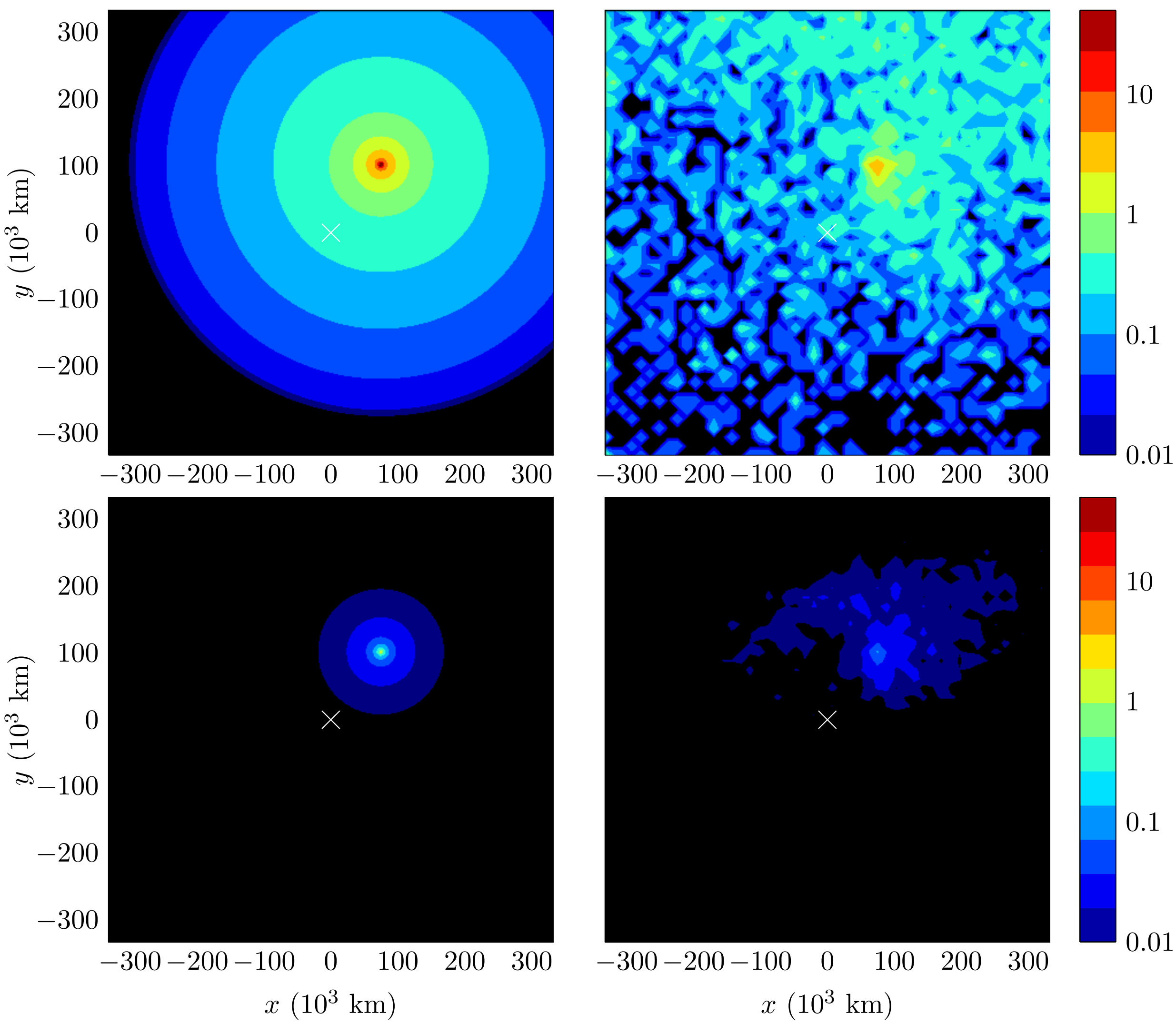

Fig. 3.— Spherical model, left, and simulation, right, renormalized to match the total number of particles in this model. The fluence of $> 4.19 \times 10^{-6}$ g particles is shown in the top plots and that of $> 4.19 \times 10^{-3}$ g particles in the bottom two plots. These sizes correspond to 100g. Colors depict the total fluence per square meter as a function of location in a plane perpendicular to the velocity vector of C/2013 A1 (Siding Spring) relative to Mars. The planet Mars is located at the origin and marked with a white X. For this plot, $\rho = 1$ g/cc and $g = 0.5$. For the spherical model, we used a coma radius of 378,000 km (see Table 2).

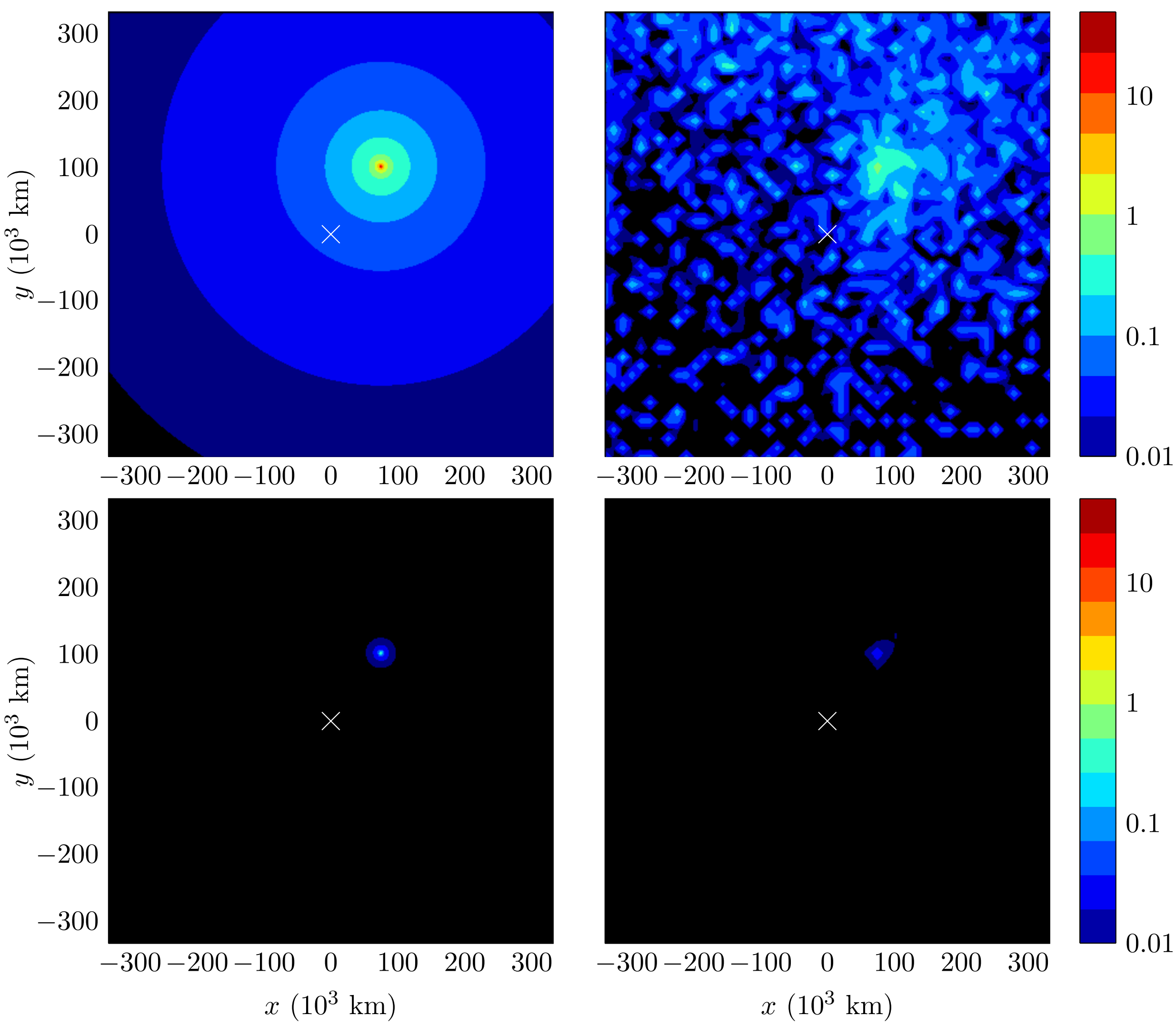

Fig. 4.— Spherical model, left, and simulation, right, renormalized to match the total number of $> 4.19 \times 10^{-6}$ g particles (top) and $> 4.19 \times 10^{-3}$ g particles (bottom) in this model. Colors depict the total fluence per square meter as a function of location in a plane perpendicular to the velocity vector of C/2013 A1 (Siding Spring) relative to Mars. The planet Mars is located at the origin and marked with a white X. For this plot, $\rho = 0.1$ g/cc and $g = 1$. For the spherical model, we used a coma radius of 814,000 km.

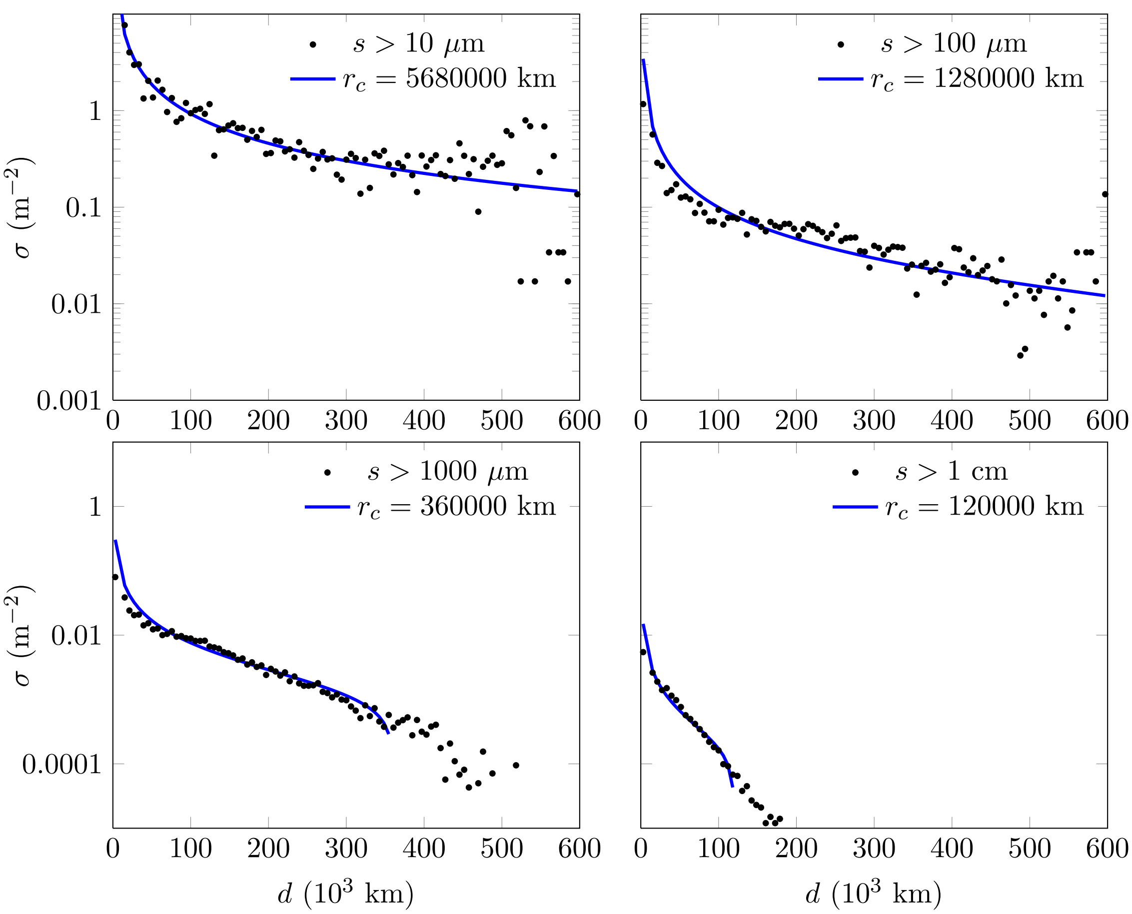

Fig. 5.— Fluence as a function of cometocentric distance, d, for simulations (black points) and spherical model (blue curve) for different size bins. We have matched our model coma radius to the simulations for each size cutoff, finding the coma radius that minimizes the chi-square. The resulting fits for coma radius are displayed in Table 2.

4. Comparison with 1P/Halley

While our goal is to quantify the risk that C/2013 A1 (Siding Spring), a long-period, Oort cloud comet, poses to Martian spacecraft, short-period comet 1P/Halley is the only comet for which detailed coma data is available. Therefore, we use 1P/Halley to test whether our model is self-consistent. For the same choice of albedo, dust size distribution, and density, the ratio of total number of particles in the coma of Siding Spring during the Mars encounter to the total number of particles in the coma of Halley during the Giotto spacecraft encounter should be equal to the ratio of the total cross section of the particles. Using 3.88 for the absolute magnitude of Halley's comet at the time of the Giotto flyby (Hughes, 1988), we obtain:

$$\begin{aligned}\frac{A_s}{A_h} &= \frac{r_s^{2-0.4\cdot \mathrm{k1}_s}}{r_h^{2-0.4\cdot \mathrm{k1}_h}} \cdot 10^{-0.4(\mathrm{M1}_s - \mathrm{M1}_h)} \\ &= \frac{(1.4 \text{ AU})^{2-0.4\cdot 11}}{(0.9024 \text{ AU})^{2-0.4\cdot 8}} \cdot 10^{-0.4(5.2-3.88)} = 0.12 \end{aligned} \tag{27}$$

We can improve upon this by using Equation 23 to calculate the fluence encountered by Giotto and comparing with observations. In its journey through Halley's coma, Giotto recorded 12,000 dust impacts[3]. It encountered the first of these particles 122 minutes before the time of closest approach; at the spacecraft's relative speed of $v_G = 68.373$ km/s (Levasseur-Regourd et al. 1986), this translates to a distance of 500,000 km from the nucleus. The dust distribution was observed to be asymmetrical; however, within 100,000 km, Fulle et al. (2000) find that the dust abundance drops off with the square of the distance, consistent with our simple model (Equation 16).

Figure 6 gives us the dust flux (which we shall denote as ξ) as a function of distance from the nucleus of 1P/Halley as recorded by Giotto's dust impact detector (DID). In order to compare with our fluence, σ, we integrate the dust flux over the spacecraft's path and normalize using Giotto's encounter velocity:

$$\sigma(b_G) = \int \xi(\vec{x}) \mathrm{d}t \tag{28}$$

$$= \int \xi(r) \frac{\mathrm{d}x/\mathrm{d}r}{\mathrm{d}x/\mathrm{d}t} \mathrm{d}r \tag{29}$$

$$= \frac{1}{v_G} \int \frac{\xi(r) \cdot r}{\sqrt{r^2 - b_G^2}} \mathrm{d}r \tag{30}$$

$$\approx \frac{2}{v_G} \int_{10^3 \mathrm{km}}^{10^5 \mathrm{km}} \xi(\vec{x}) \mathrm{d}x \,,$$

where $b_G = 600$ km is the close-approach distance between Giotto and 1P/Halley. Using this approach, we obtain a total of 13358 particles encountered per square meter of spacecraft along Giotto's trajectory.

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[3] `http://sci.esa.int/jump.cfm?oid=31878`

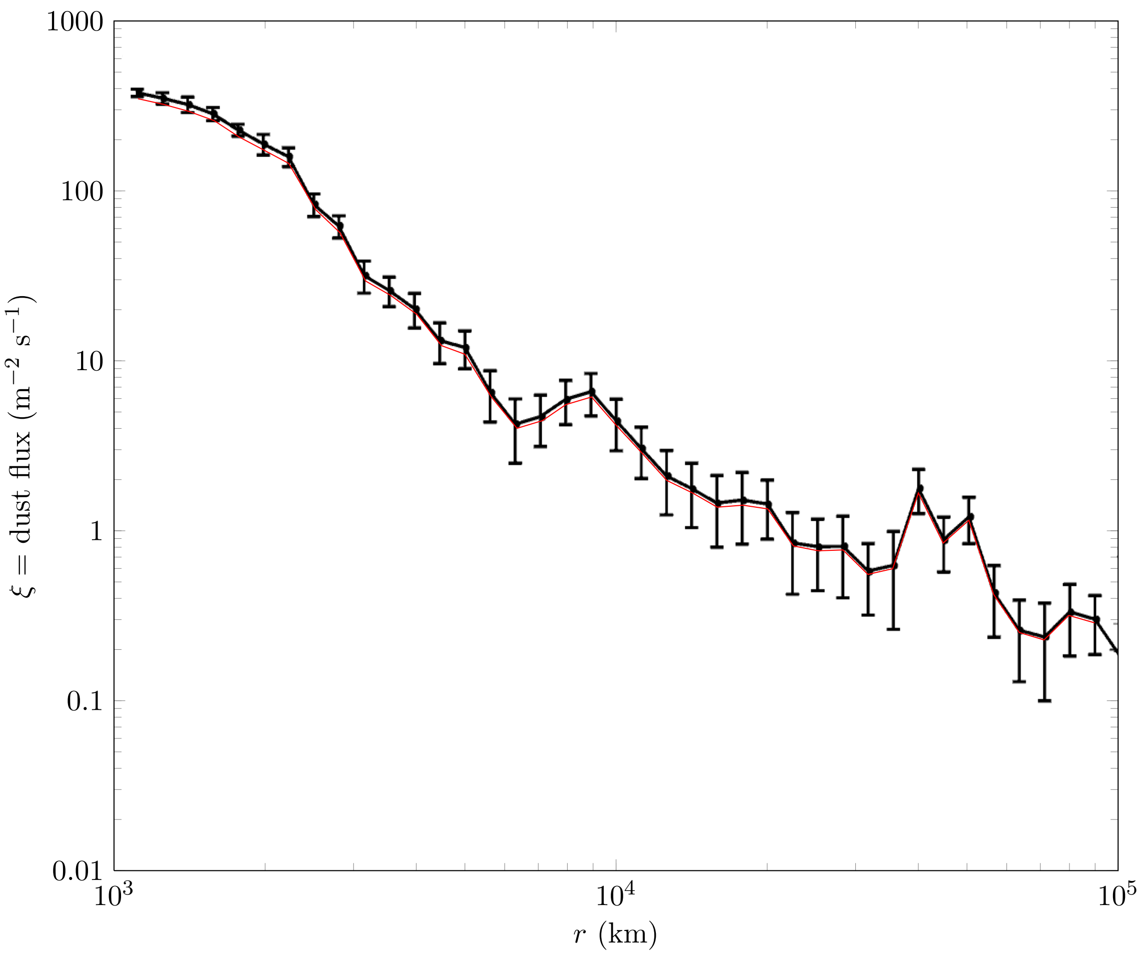

Fig. 6.— Dust flux as a function of cometocentric distance, reproduced from Fulle et al. (2000). Black data points and error bars are from Fulle et al. (2000); the red curve is our reproduction. We extract and integrate these data in order to determine the total fluence encountered by the Giotto spacecraft in its passage through the coma of 1P/Halley.

In comparison, Equation 23 produces:

$$\sigma \approx 14748 \text{ m}^{-2} f_k \cdot g \cdot (0.9023)^{-(\beta-1.2)}$$
$$\left(\frac{a}{0.04}\right)^{-1} \left(\frac{\rho}{0.1 \text{ g/cc}}\right)^{\frac{2}{3}} \left(\frac{r_c}{200{,}000 \text{ km}}\right)^{-1} 10^{-0.4(\mathrm{M1}-3.88)} . \quad (31)$$

(See also Figure 7). While particles were detected at more than twice our “best” radius of $r_c =$ 200,000 km, the distribution was observed to be asymmetrical and the relative error on the flux distribution at large radii is quite large. Thus, we consider our model to have successfully (i.e., within an order of magnitude) reproduced the dust flux encountered by Giotto near Halley. Note that in this case, we have calculated the total flux of all dust particles within the range 7.71×10^{-9} g - 31.0 g, while for C/2013 A1 we computed the flux only of potentially hazardous large ($> 4 \times 10^{-6}$g) particles.

5. Conclusions

We have developed an analytic model of the dust abundance in cometary comae that can be used to obtain order-of-magnitude estimates of impact risk. This model relies on observables such as total cometary magnitude to estimate the brightness of the coma; this brightness is then compared with typical dust properties to generate a dust distribution. Finally, integration along a trajectory yields a total fluence of particles, which, for small values, is approximately the risk of impact.

This model can be applied to comet C/2013 A1 (Siding Spring), which is projected to make a close approach to Mars on October 19, 2014. The close approach distance, 113,000 km, is sufficiently small that the comet is likely to engulf Mars and its natural and man-made satellites in the coma. We use the close approach distance and cometary magnitude to model the fluence of larger than 100 micron particles at Mars during the encounter. We find that while the exact number of expected impacts varies with comet and dust properties, the total fluence could be as large as 0.2 impacts per square meter.

We rely on studies of dust impact data recorded by the spacecraft Giotto on its route through comet 1P/Halley's coma to constrain dust properties, but not dust quantity. To check for self-consistency, we model the coma of 1P/Halley itself and extract the fluence along Giotto's trajectory. Our result agrees with the recorded fluence at the order-of-magnitude level.

We also check our assumptions regarding the spatial distribution (i.e., that it is spherically symmetric) by comparing our spherical model with simulations. We find that a coma radius of several hundred thousand kilometers best describes these results, although the extent of the coma is a function of the duration of activity and the particle ejection velocity, which in turn depends on quantities such as particle density and nucleus size.

Comets are notoriously unpredictable; the magnitude of C/2013 A1 may very well change

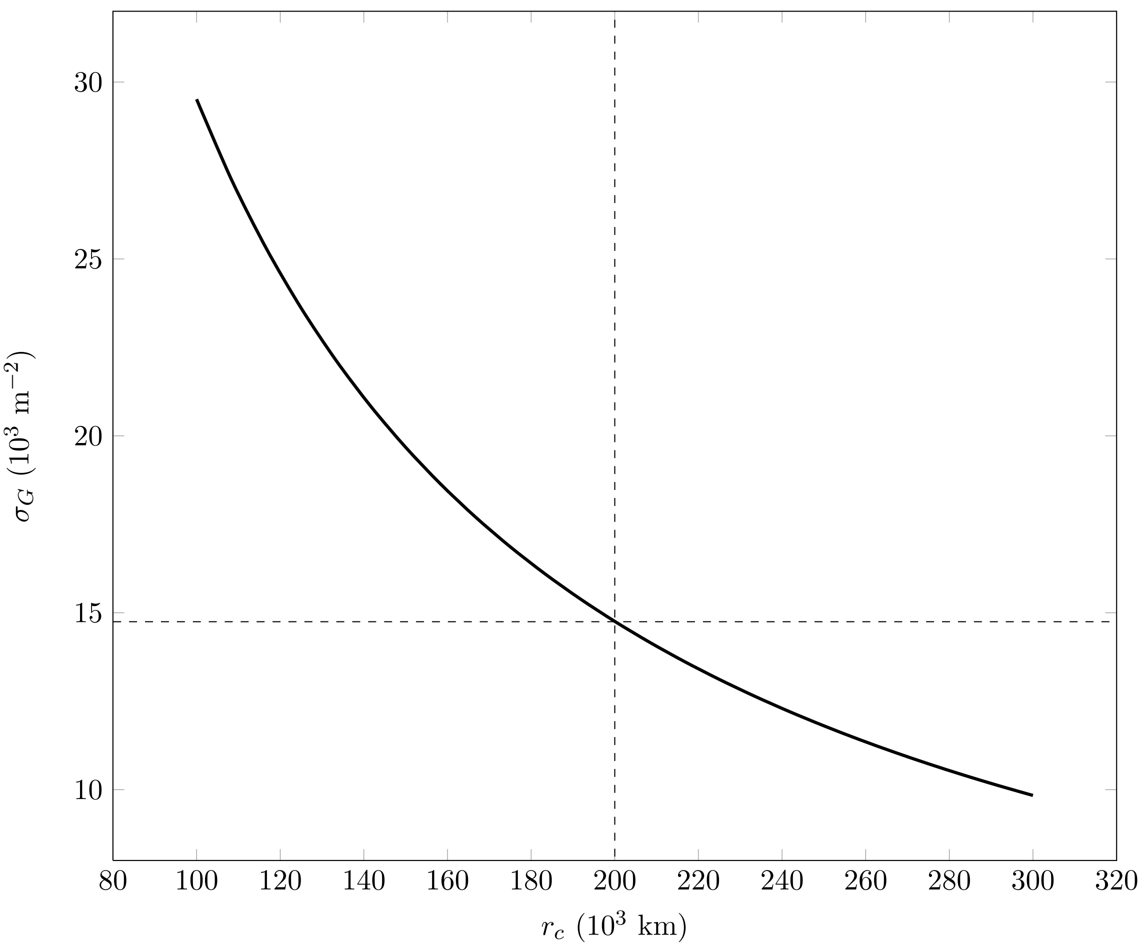

Fig. 7.— Total fluence as a function of comet radius as calculated using our model for spacecraft Giotto passing through the coma of 1P/Halley. Fluence drops off with comet radius (see Equation 23), and has a value comparable to that encountered by Giotto (i.e., 14,000) for a comet radius of 180,000 - 200,000 km.

significantly as it moves inward through the solar system. Additionally, neither the start of activity nor the size of the nucleus has been well constrained. Thus, we have expressed our results in parametrized form throughout this analysis, and our estimates can thus be easily updated as additional observations of C/2013 A1 (Siding Spring) are made.

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A. Asteroidal Magnitudes

As a check, we apply Equation 7 to a special case: asteroids, like comets, reflect solar radiation, but do not produce a coma. Hence, $\beta = 0$ in this limiting case. Furthermore, the area is simply the cross-sectional area of the asteroid, $\pi d^2/4$. In Figure 8, we plot the magnitudes of asteroids from JPL's small body database for which magnitudes, albedos, and diameters have been measured. Note the close agreement between measured and calculated values of H.

We can also rearrange Equation 7 to obtain the following relation for diameter:

$$d^2 = (1343 \text{ km})^2 \cdot 10^{-0.4H}/a\,. \tag{A1}$$

Note that this is a close match to the traditional 1329^2 figure (see, for example, Jedicke 1998).

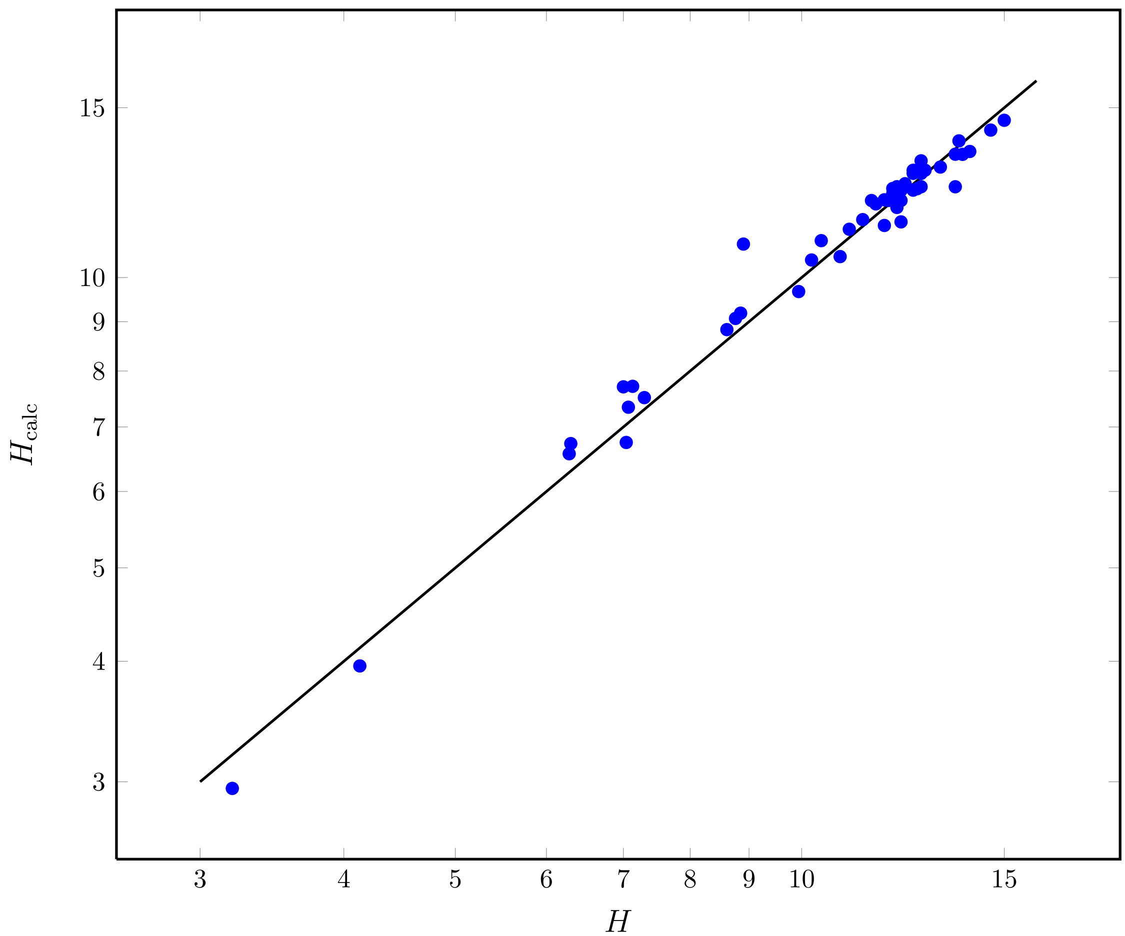

Fig. 8.— Magnitude calculated from asteroid diameter and albedo ($H_{\rm calc}$) using Equation 7 versus actual magnitude (H) for asteroids pulled from JPL's small body database. Note the resemblance between the data and the expected $H_{\rm calc} = H$. We have excluded asteroids for which diameter, magnitude, or albedo is unknown, and asteroids for which albedo has been calculated from H and d or vice versa (and which lie exactly on the line above).

B. Subradiant Location

The Martian latitude and longitude of the subradiant point of the meteor shower accompanying C/2013 A1 (Siding Spring) can be computed from the velocity vector of the comet relative to Mars. According to JPL Horizons, the time of closest approach is Julian Date 2456950.285, at which the velocity of C/2013 A1 in the Mars-centered inertial frame is :

$$\vec{v}_{\rm ss} = (-53.56, 13.98, 8.23)\ \text{km/s} \tag{B1}$$

We can determine the latitude of the subradiant point, ϕ, as follows:

$$\sin\phi = \frac{-v_z}{\sqrt{v_x^2 + v_y^2}} = -0.149 \tag{B2}$$

Because $-\pi/2 < \phi < \pi/2$, latitude is fully determined by the above equation and $\phi = -8.55°$.

To obtain the longitude, we extract the position of Mars's body center relative to a point at latitude 0, longitude 0 on its surface:

$$\vec{r}_{\rm Mars} = (-3364.8, -460.82, 0)\ \text{km} \tag{B3}$$

Next, we use the dot product of the two to determine longitude, λ:

$$\cos\lambda = \frac{\vec{v}_{xy} \cdot \vec{r}_{\rm Mars}}{|\vec{v}_{xy}||\vec{r}_{\rm Mars}|} = 0.924 \tag{B4}$$

Evaluation of the above equation and examination of the vectors yields a latitude of $\lambda = 22.4°$ W.

Finally, we use the equations of Allison & McEwen (2000) as implemented in NASA's Mars24 Sunclock[4] to determine the local mean solar time (LMST) at the given latitude and longitude during the time of close encounter, which is 5:30.

[4] `http://www.giss.nasa.gov/tools/mars24/`