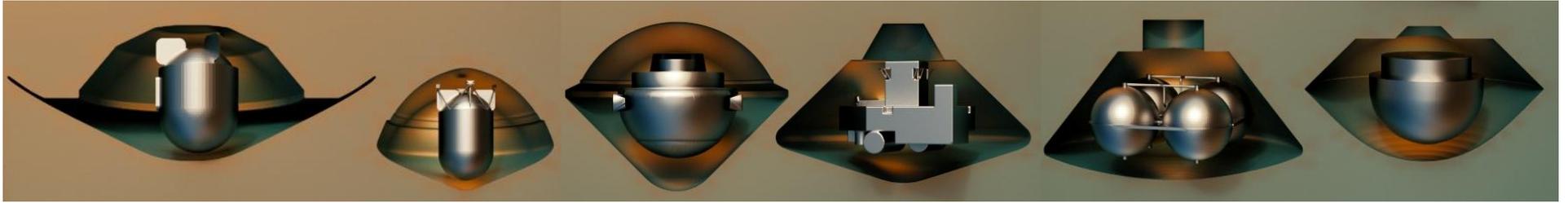




# INTRODUCTION TO TRAJECTORIES

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*Entry, Descent, and Landing Systems Short Course  
10<sup>th</sup> International Planetary Probe Workshop  
San Jose, California  
June 15-21, 2013*



# Entry, Descent, and Landing Systems Short Course

Subject: Introduction to Trajectories

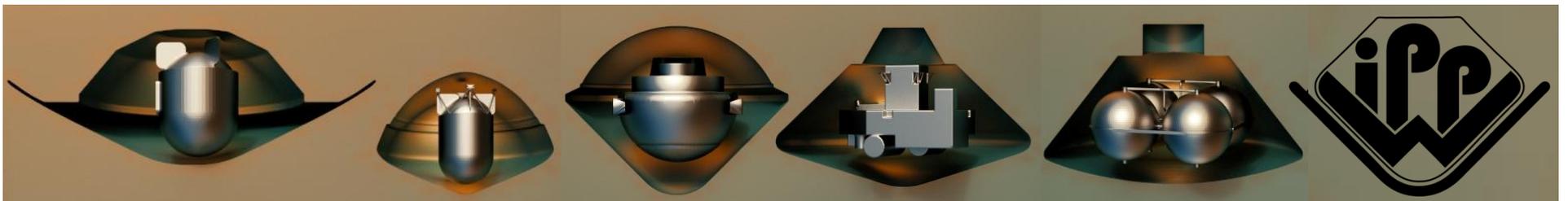
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*NASA Langley Research Center*

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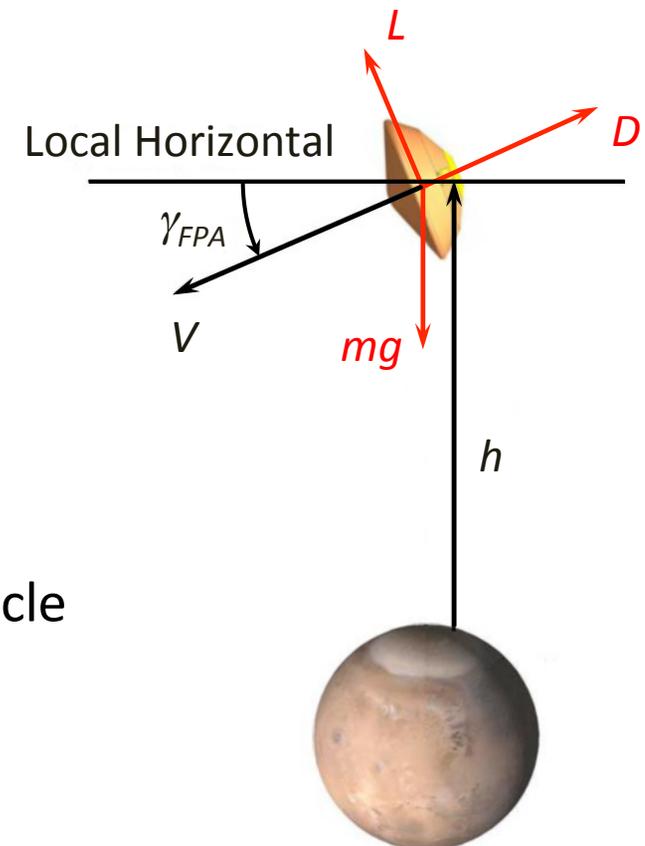
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# Definition of Trajectory

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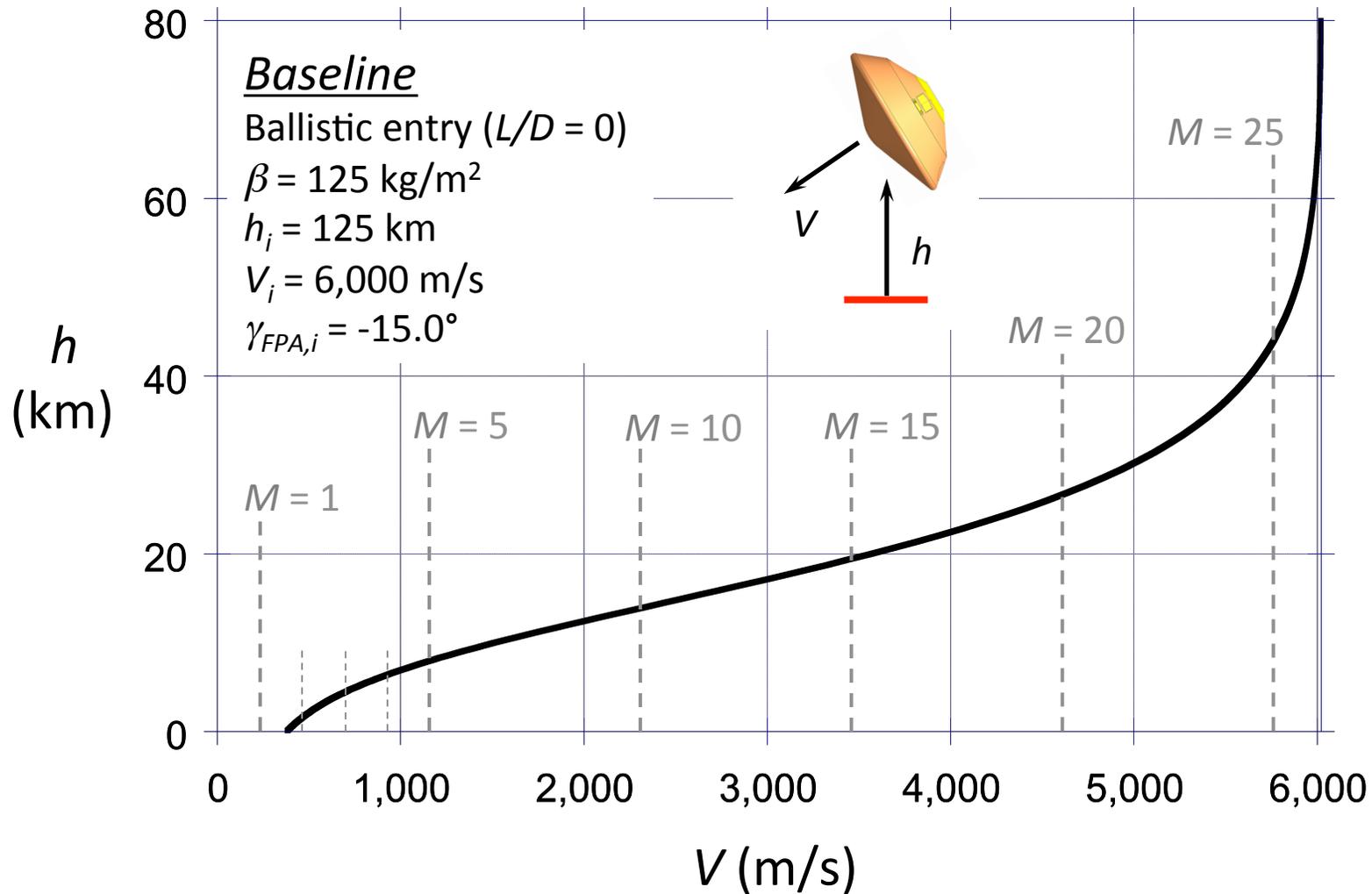
## Trajectory

- The flight path taken by an entry vehicle
- Described by its state variables
  - latitude and longitude
  - azimuth (flight direction wrt. north)
  - altitude,  $h$
  - velocity,  $V$
  - flight path angle,  $\gamma_{FPA}$
- Driven by the forces acting on the entry vehicle
  - aerodynamic
    - lift,  $L$  and drag,  $D$
  - gravity,  $mg$
  - propulsive

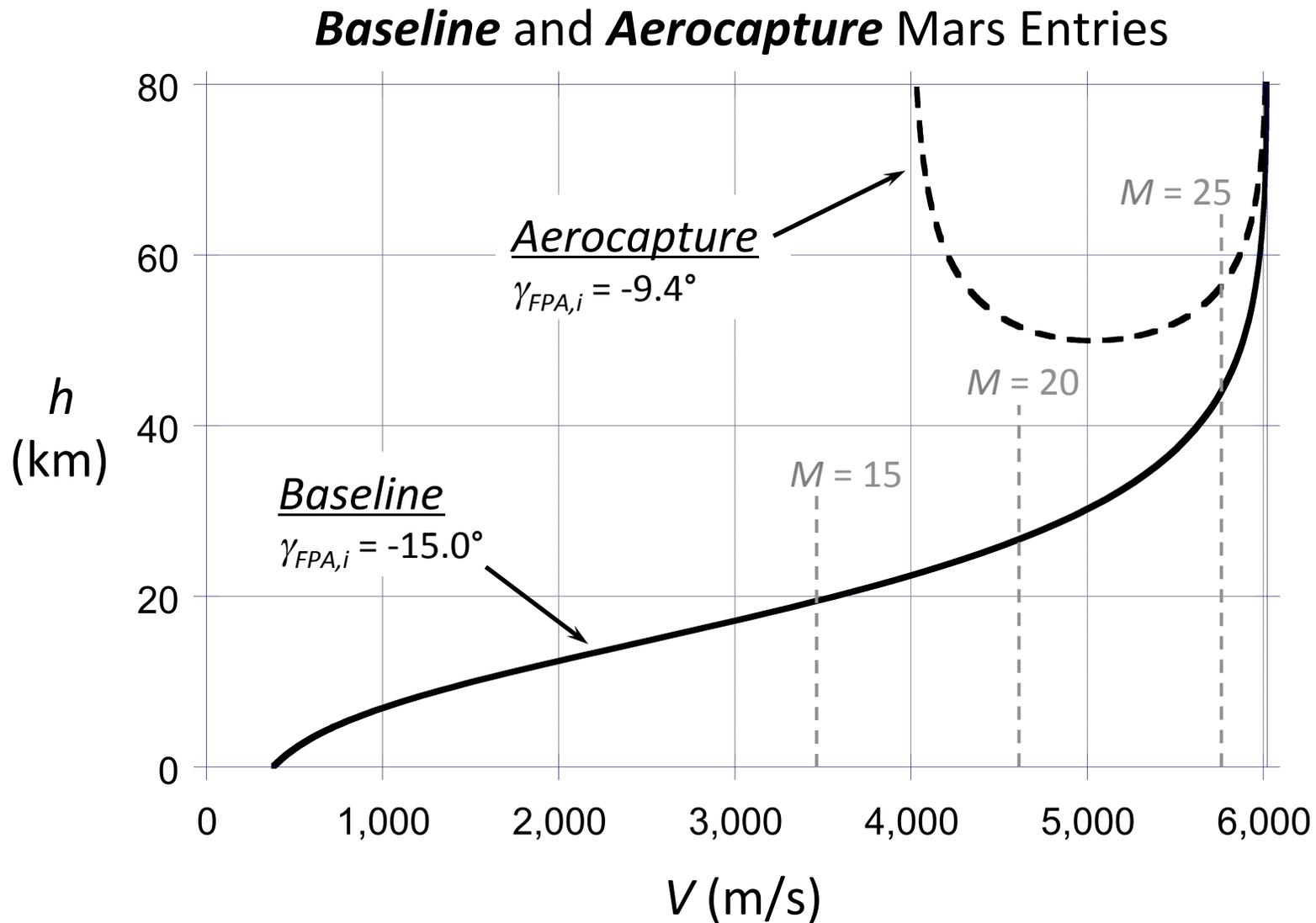


# Velocity-Altitude Diagram

## Baseline Mars Entry



# Velocity-Altitude Diagram



# Energy and Aeroheating

---

Energy and aeroheating will be described in terms of the following three quantities:

$E$  = **Total Energy** metric

= (Kinetic Energy + Potential Energy)<sub>Nondimensionalized</sub>

Nondimensionalized wrt. the Total Energy of the *Baseline Mars Entry* at atmospheric interphase

$\dot{Q}$  = Aerothermodynamic convective **heat rate** metric,  $\propto \rho^{\frac{1}{2}}V^3$

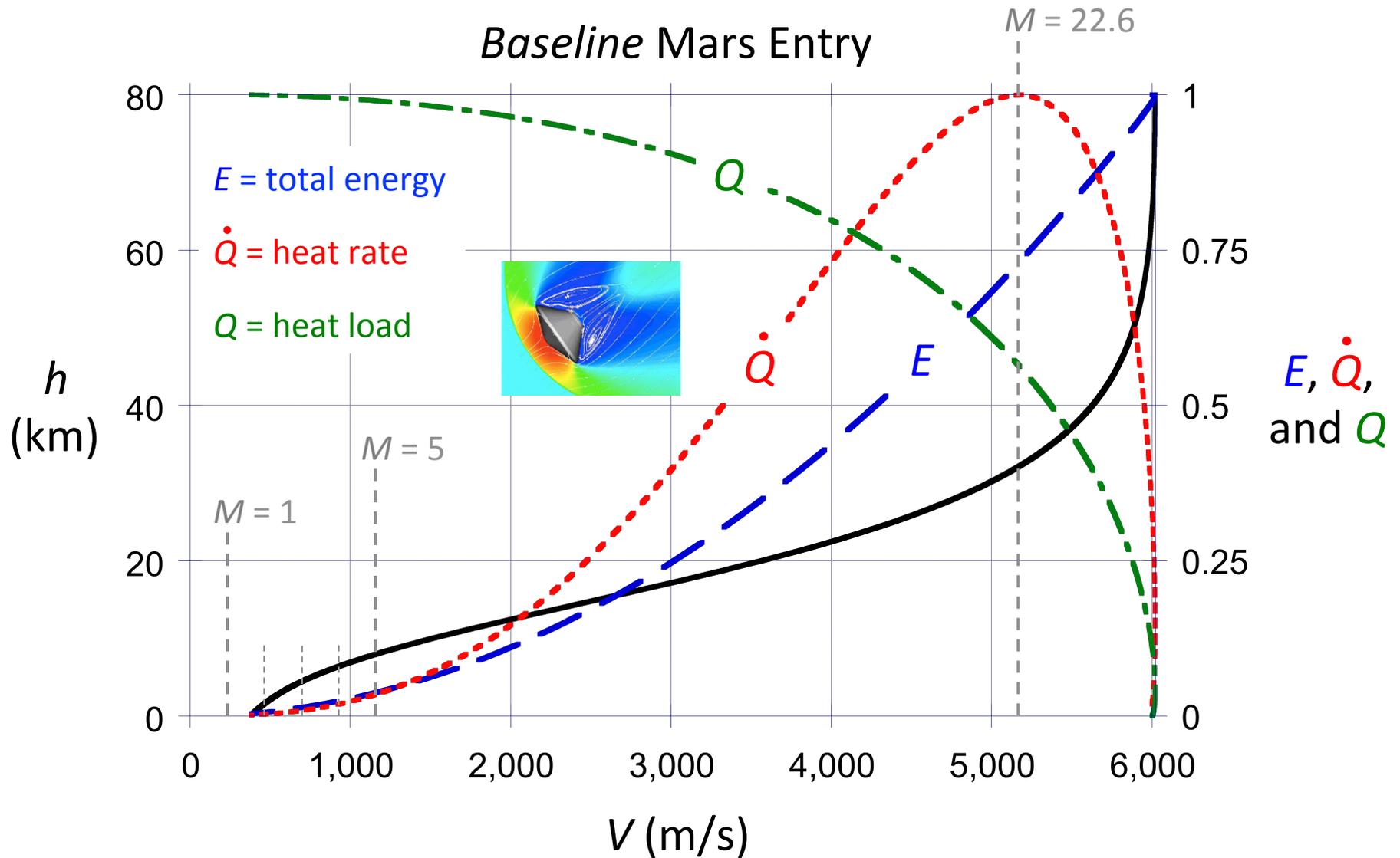
Nondimensionalized wrt. the maximum convective heat rate of the *Baseline Mars Entry*

$Q$  = Aerothermodynamic convective **heat load** metric, integration wrt. time of  $\dot{Q}$  starting at entry interphase,  $\propto \int_{t_i}^t \dot{Q} dt$

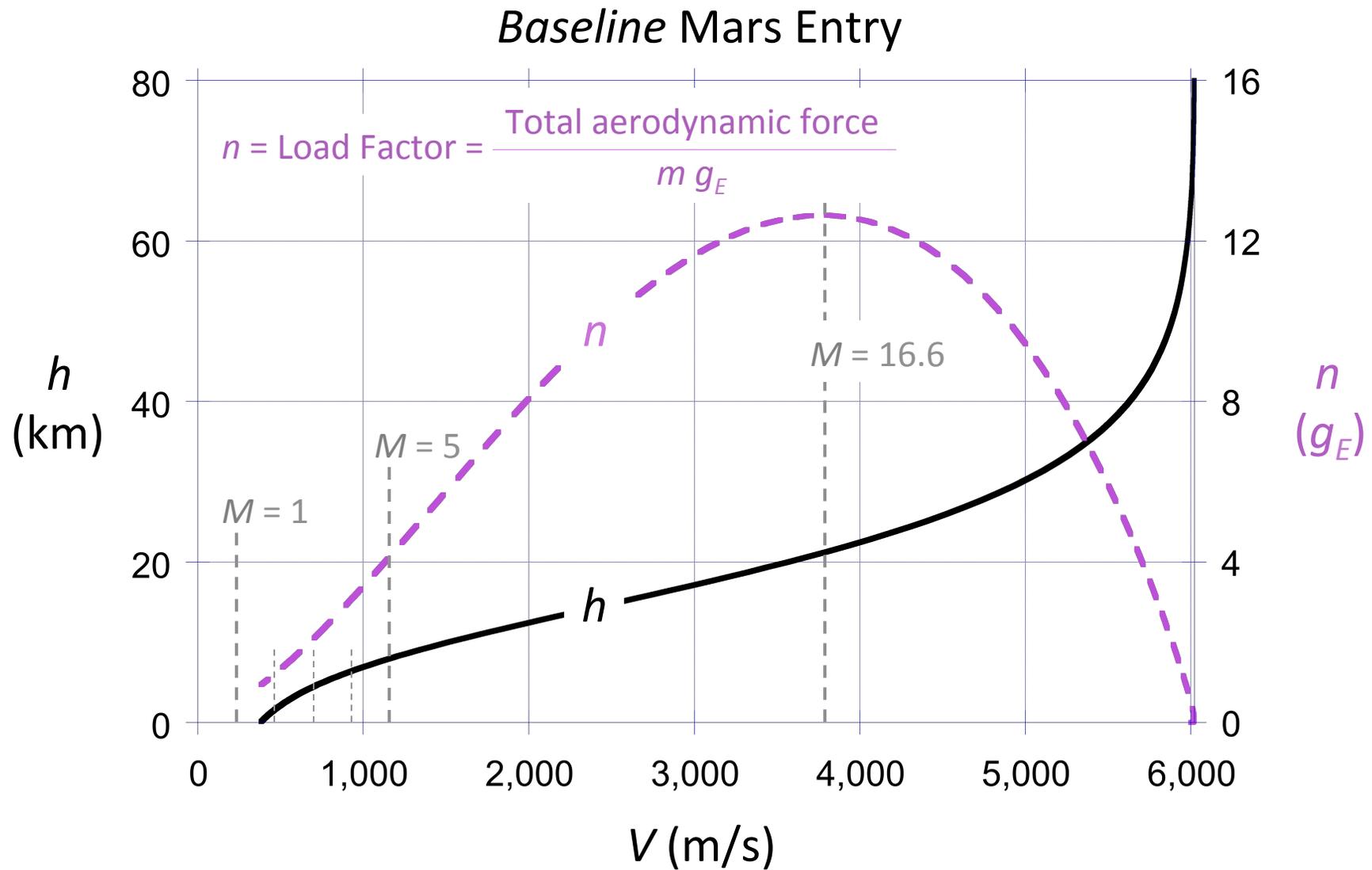
Nondimensionalized wrt. the convective heat load of the *Baseline Mars Entry* at landing

Equations for these quantities are shown on slides 34 and 35.

# Energy and Aeroheating



# Deceleration



# Key Factors Affecting the Trajectory – Entry Vehicle

---

## Ballistic Coefficient, $\beta$

A measure of the entry vehicle aerodynamic loading

$$\beta = \frac{\text{Vehicle Mass}}{\text{Drag Coefficient} \cdot \text{Aero Reference Area}} = \frac{m}{C_D A}$$

The drag coefficient,  $C_D$ , will principally depend on the shape of the entry vehicle and the Mach number,  $M$

Dimensions: [Mass/Length<sup>2</sup>]

Typical Unit: kg/m<sup>2</sup>

For the *Baseline Mars Entry* example in this presentation,  
 $\beta = 125 \text{ kg/m}^2$

# Key Factors Affecting the Trajectory – Entry Vehicle

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## Lift to Drag Ratio, $L/D$

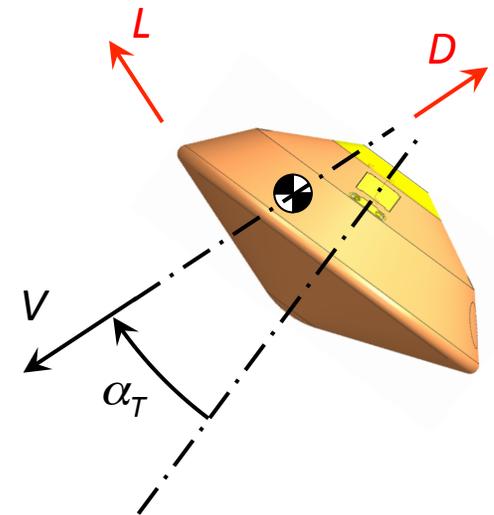
A measure of the lift efficiency of an entry vehicle

A **ballistic** entry means that  $L/D = 0$ . The *Baseline Mars Entry* example in this presentation is ballistic.

---

## Generating Lift with an Axisymmetric Entry Vehicle

- An axisymmetric vehicle with its center of mass on its axis of symmetry will trim at zero degrees total angle of attack,  $\alpha_T$ . At  $\alpha_T = 0^\circ$  it will have drag, but no lift. Thus,  $L/D = 0$ .
- Moving the center of mass off the axis of symmetry will cause the vehicle to trim at  $\alpha_T \neq 0^\circ$ .
- At  $\alpha_T \neq 0^\circ$  the entry vehicle will generate lift. Thus,  $L/D \neq 0$ . Note: Nose down yields lift up – it is not a wing!
- For a given off-axis position of the center of mass, lift will be in a specific direction with respect to the entry vehicle. The direction of this lift with respect to the trajectory can be controlled by rolling the entry vehicle.



# Key Factors Affecting the Trajectory – Initial Conditions

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Also influencing the trajectory are the initial conditions:

At an initial altitude,  $h_i$

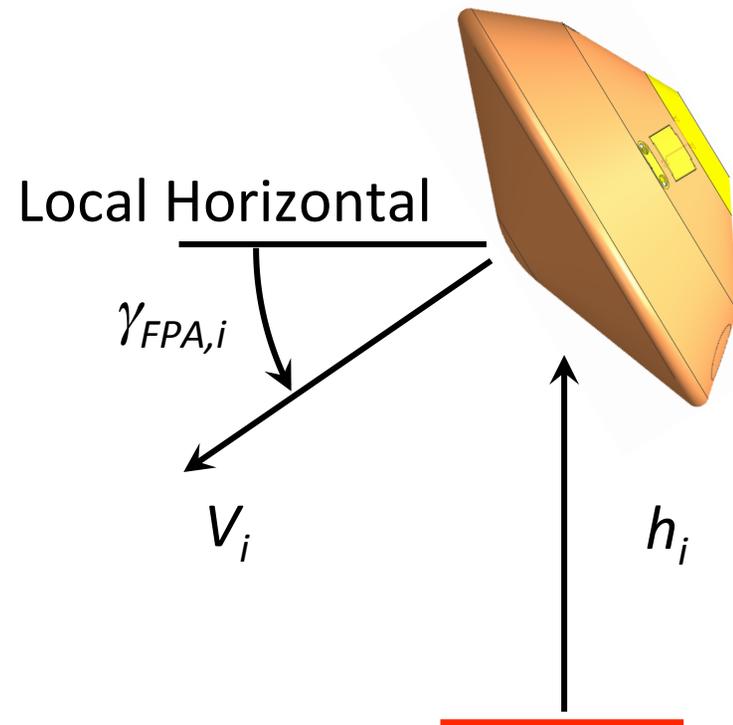
- initial velocity,  $V_i$
- initial flight path angle,  $\gamma_{FPA,i}$  (negative as shown)

For the *Baseline Mars Entry* in this presentation:

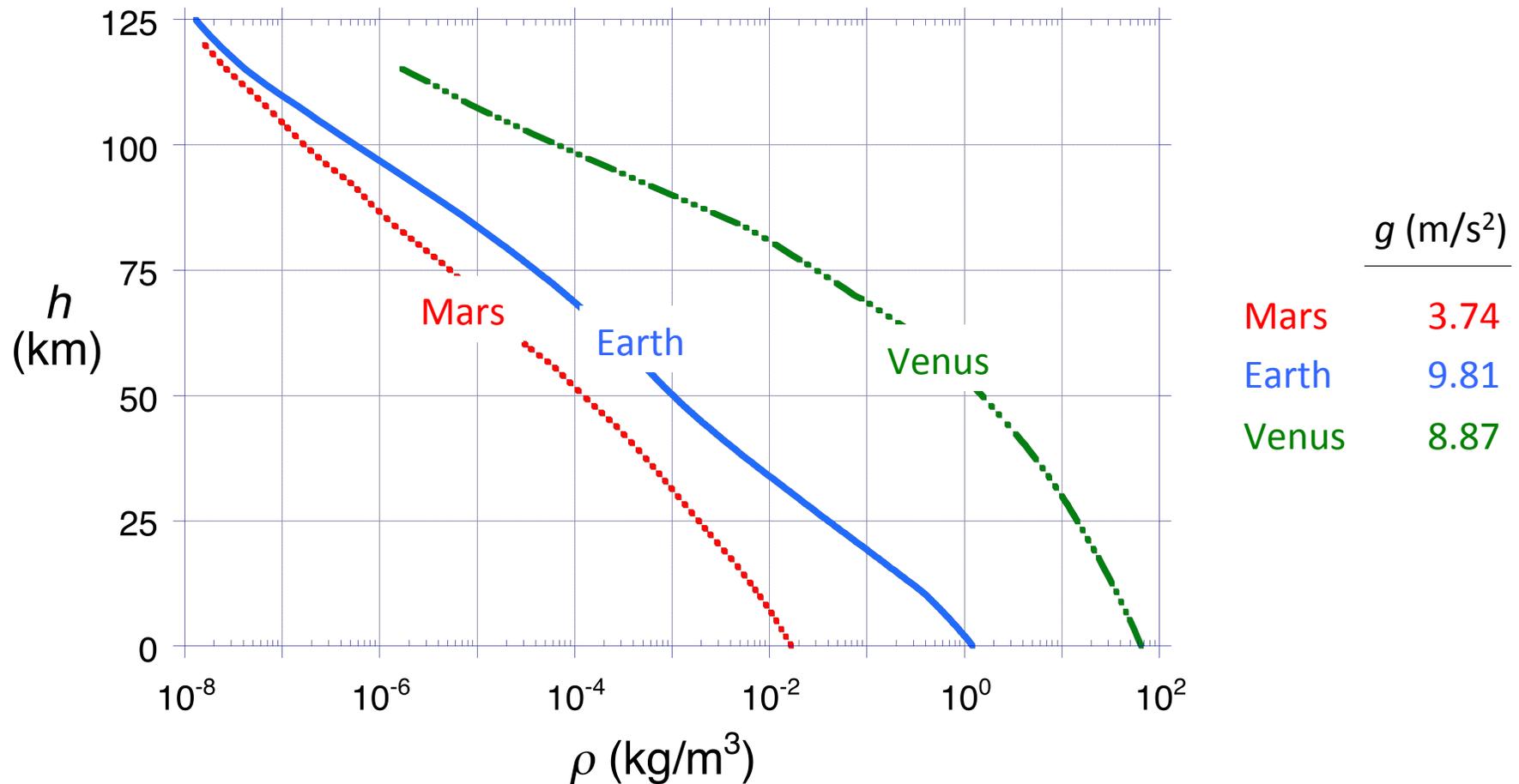
$$h_i = 125 \text{ km}$$

$$V_i = 6,000 \text{ m/s}$$

$$\gamma_{FPA,i} = -15^\circ$$

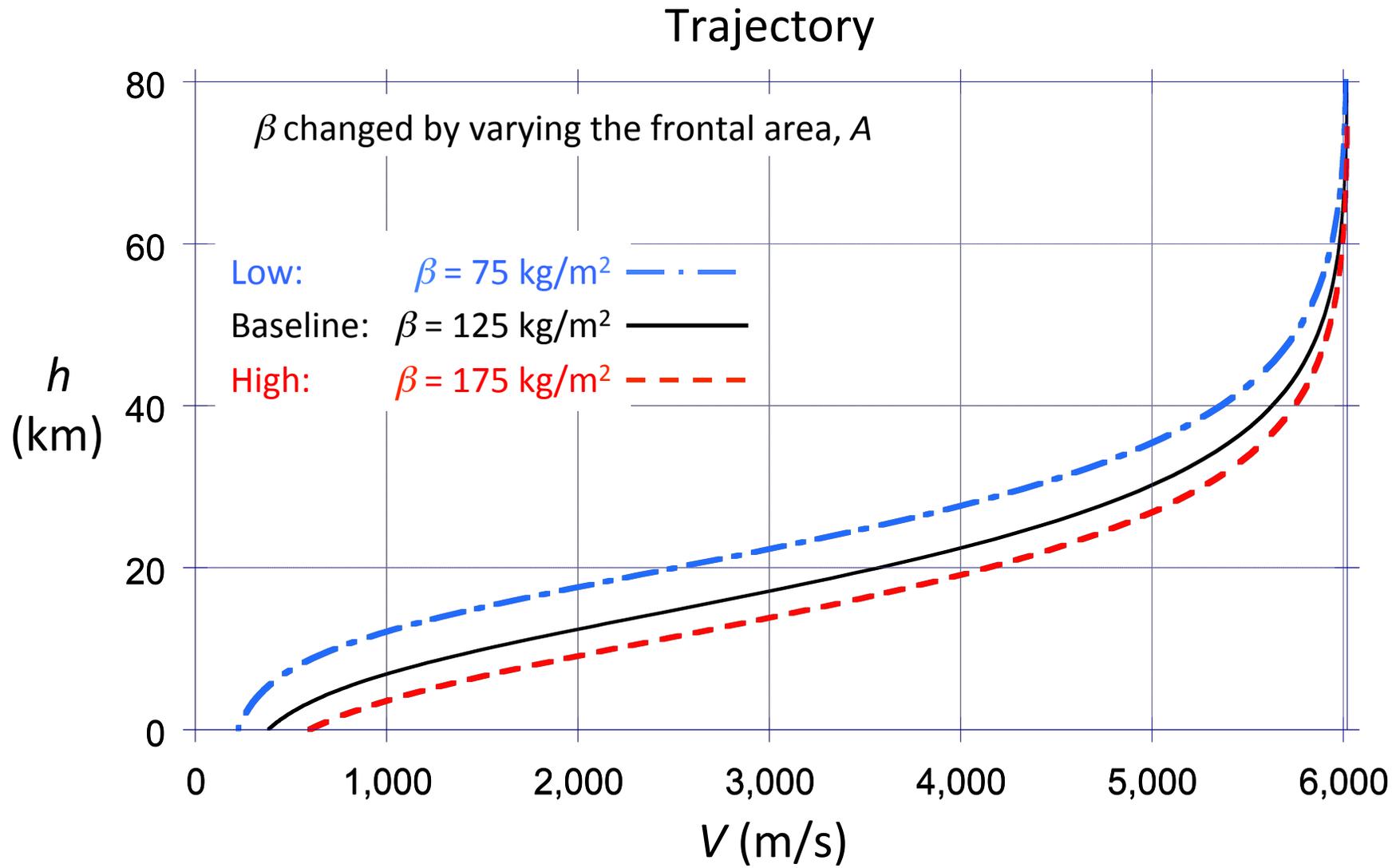


# Key Factors Affecting the Trajectory – Atmosphere and Gravity

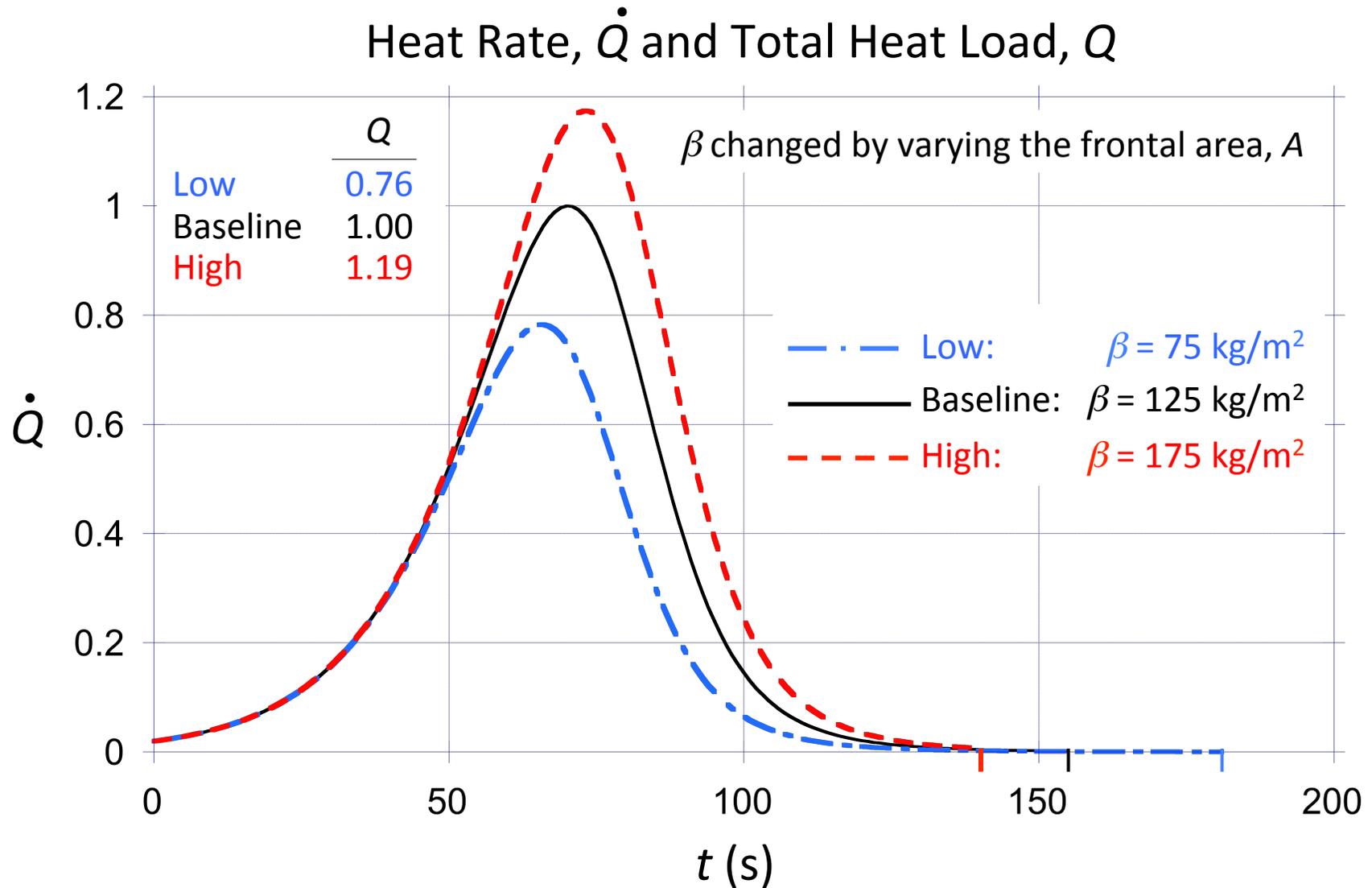


- NOAA, NASA, and USAF: U.S. Standard Atmosphere, 1976, NASA TM X-74335, 1976.
- Seiff, A. et al.: Measurements of Thermal Structure and Thermal Contrasts in the Atmosphere of Venus and Related Dynamical Observations: Results from the four Pioneer Venus Probes, *Journal of Geophysical Research*, Vol. 85, No. A13, pp. 7903-7933, 1980.
- Seiff, A. and Kirk, D. B.: Structure of the Atmosphere of Mars in Summer at Mid-Latitudes, *Journal of Geophysical Research*, Vol. 82, No. 28, 1977.

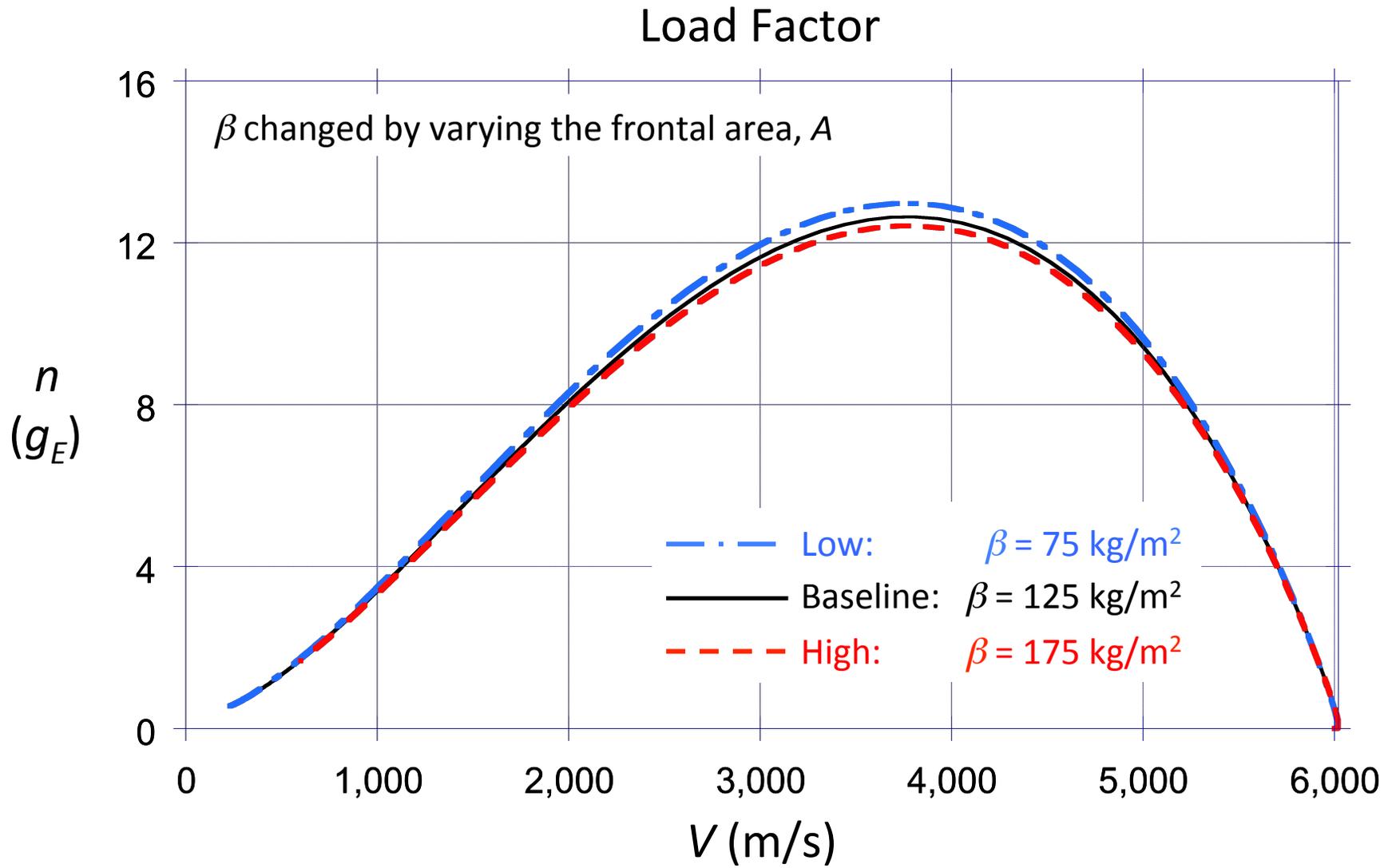
# Effect of Ballistic Coefficient



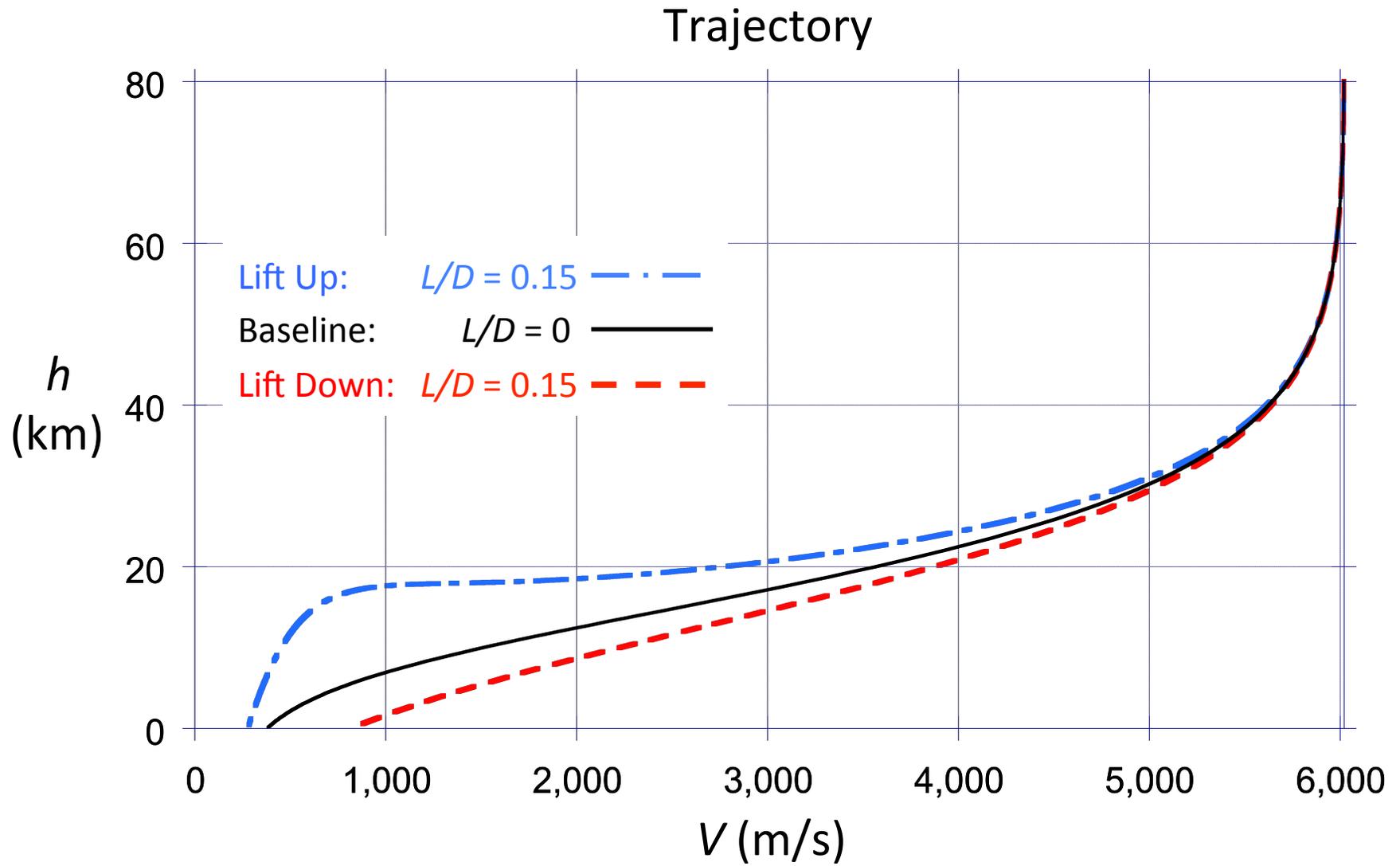
# Effect of Ballistic Coefficient



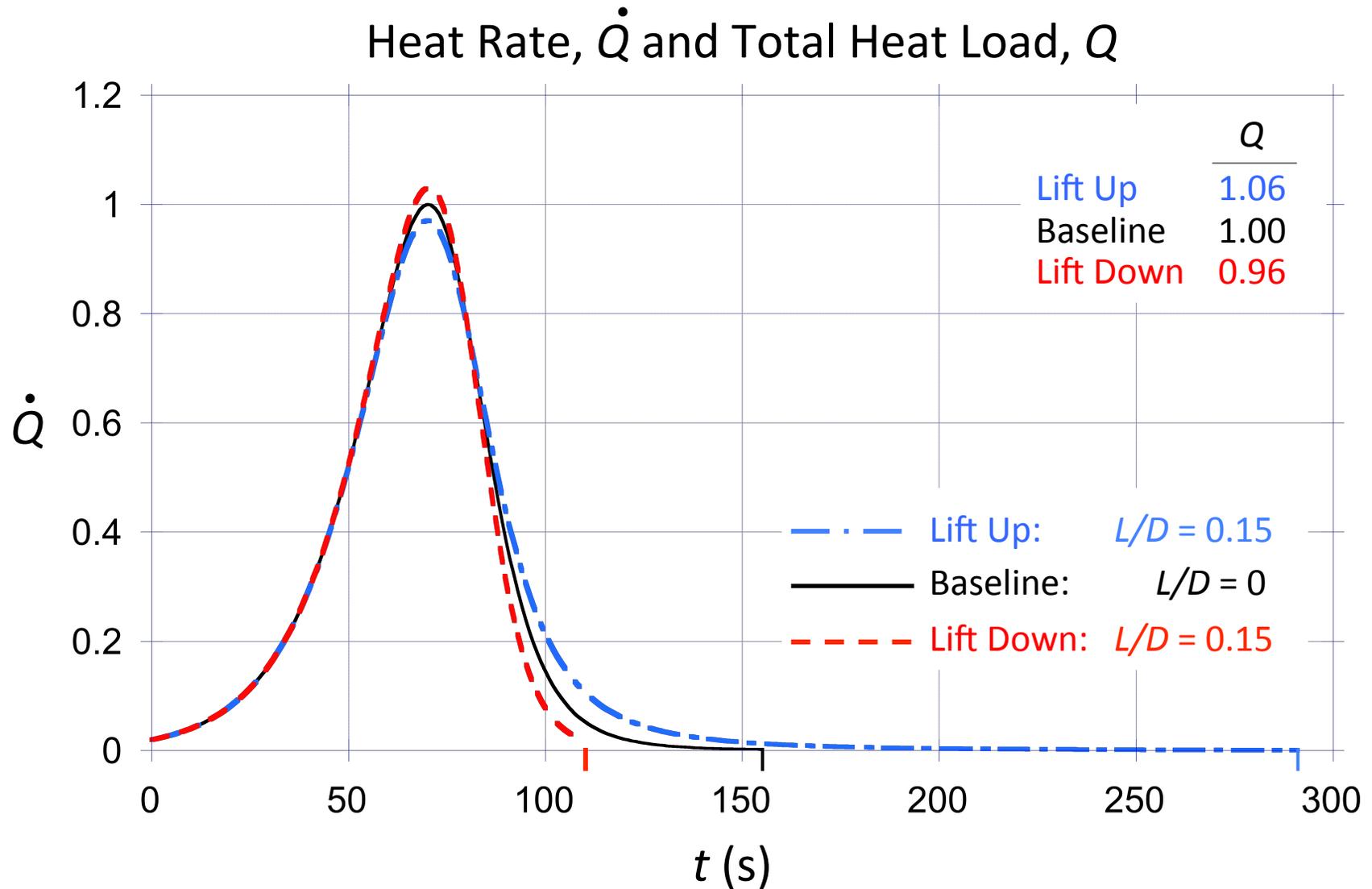
# Effect of Ballistic Coefficient



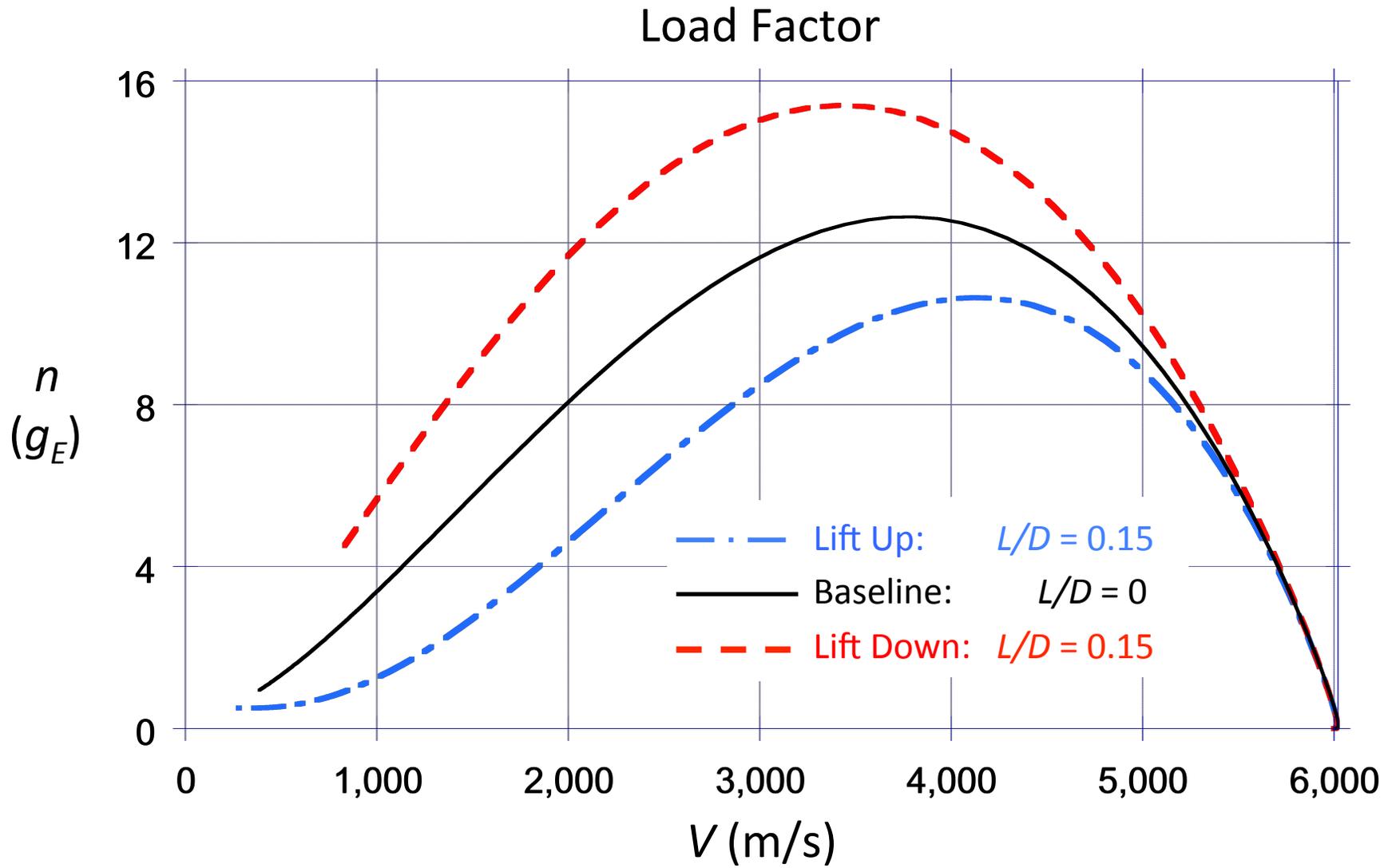
# Effect of Lift



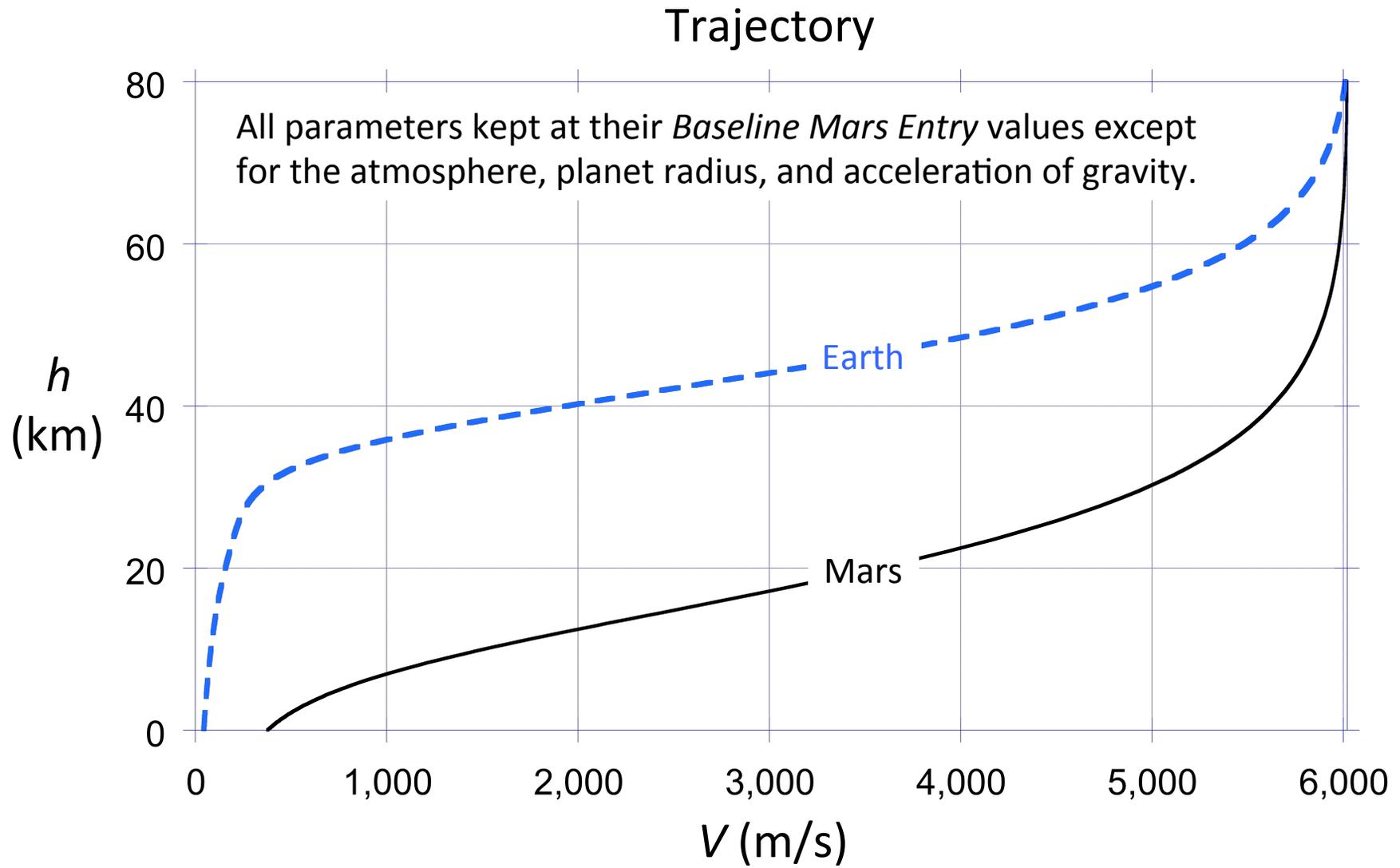
# Effect of Lift



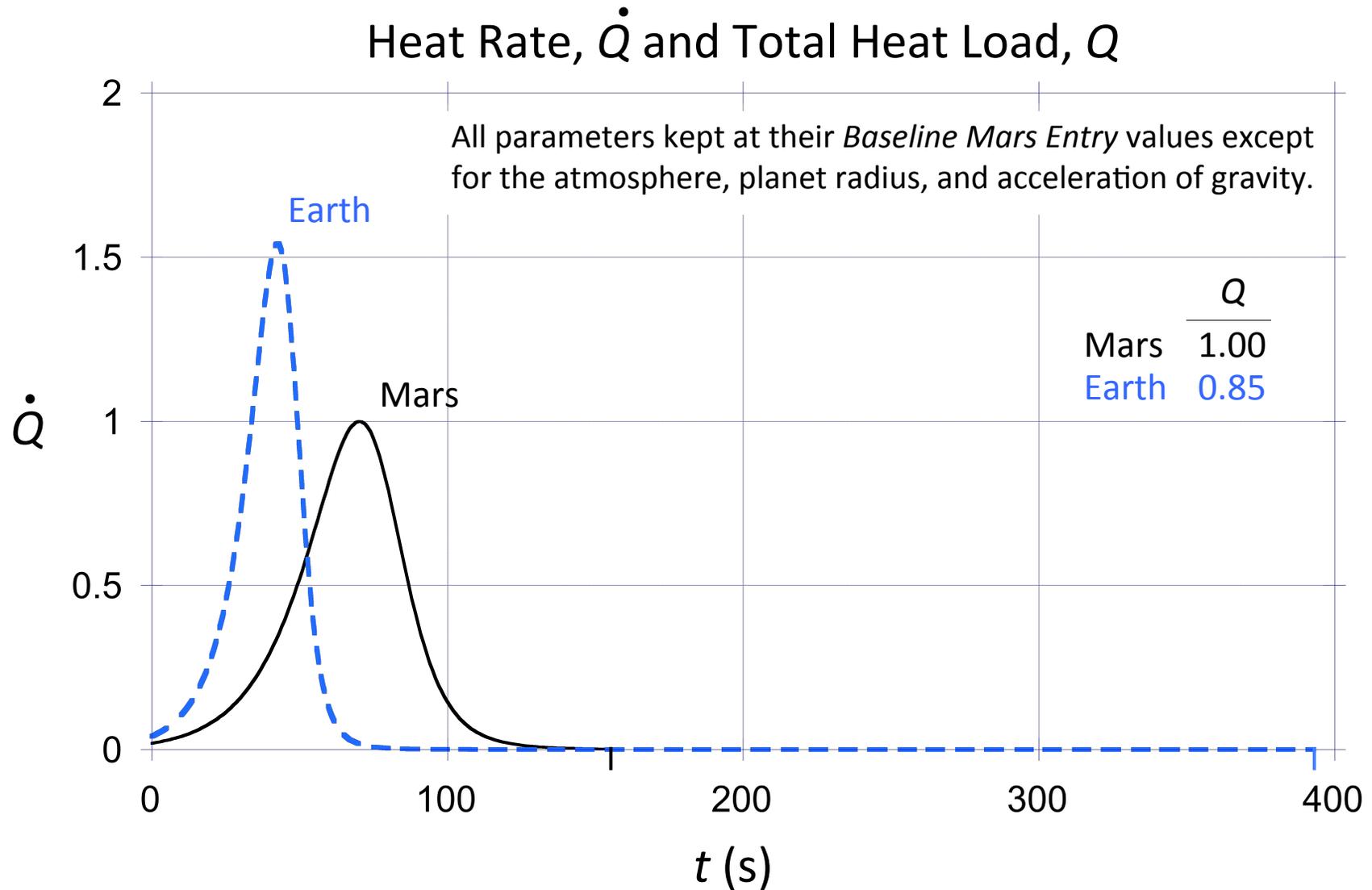
# Effect of Lift



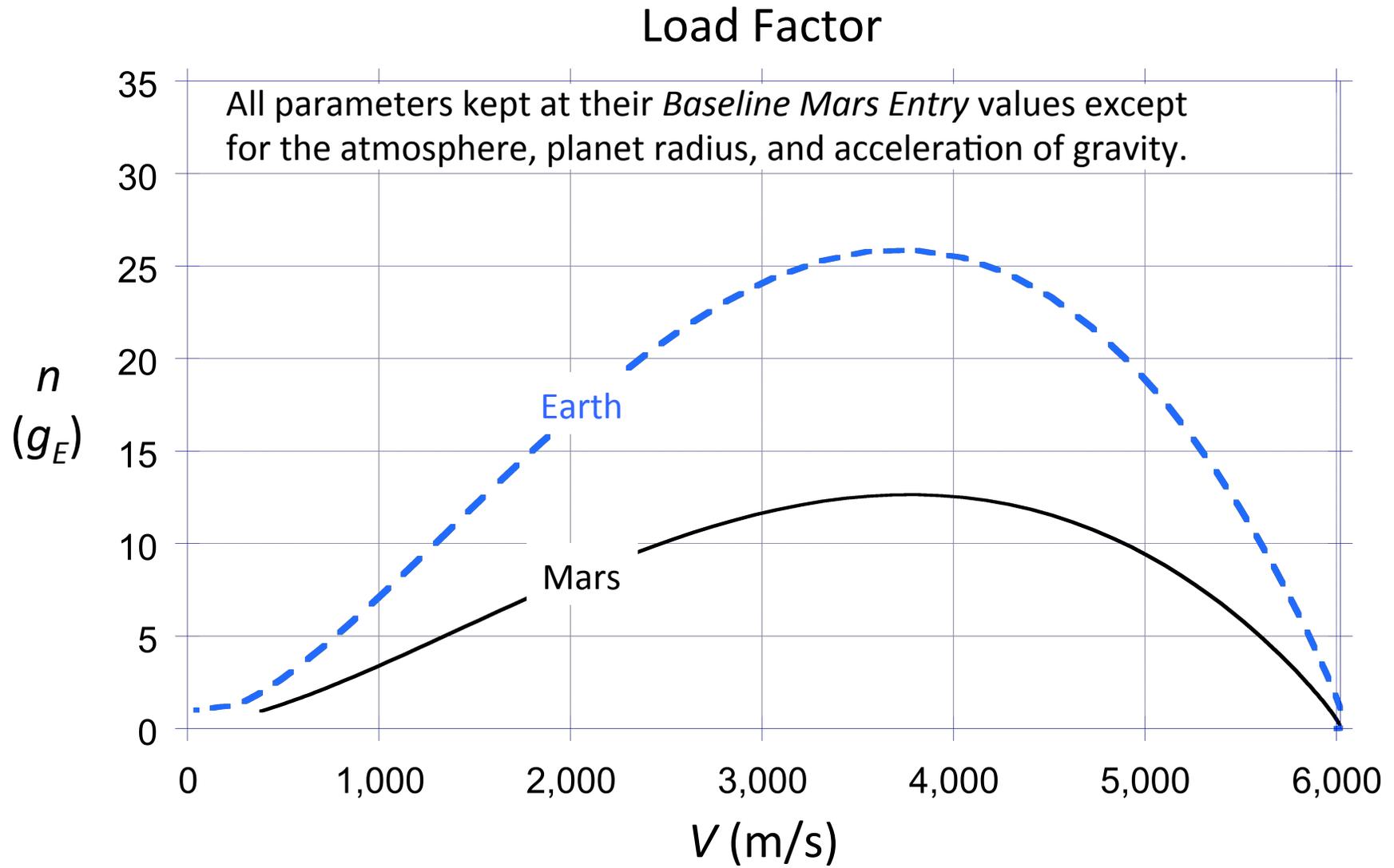
# Effect of Planet's Atmosphere and Gravity



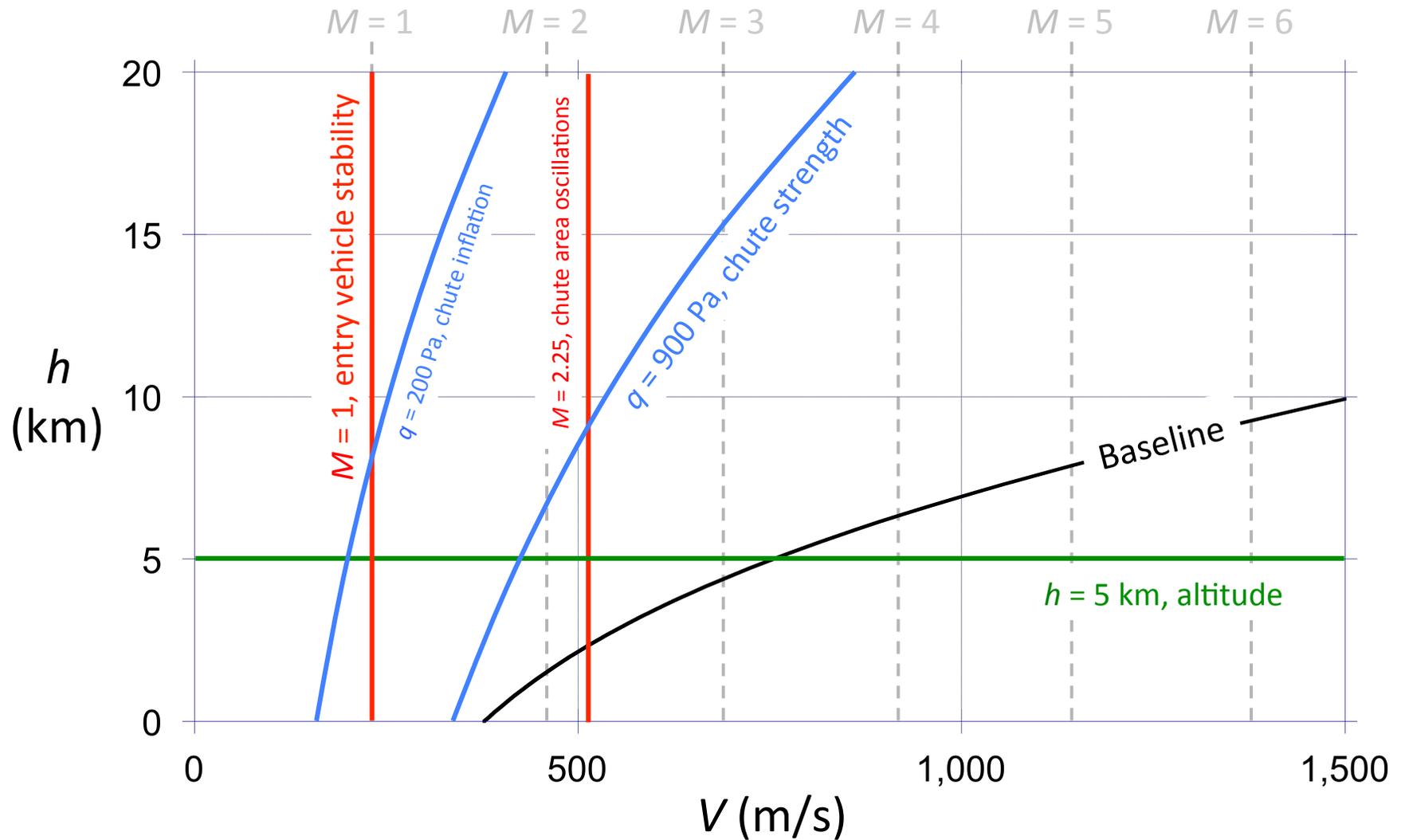
# Effect of Planet's Atmosphere and Gravity



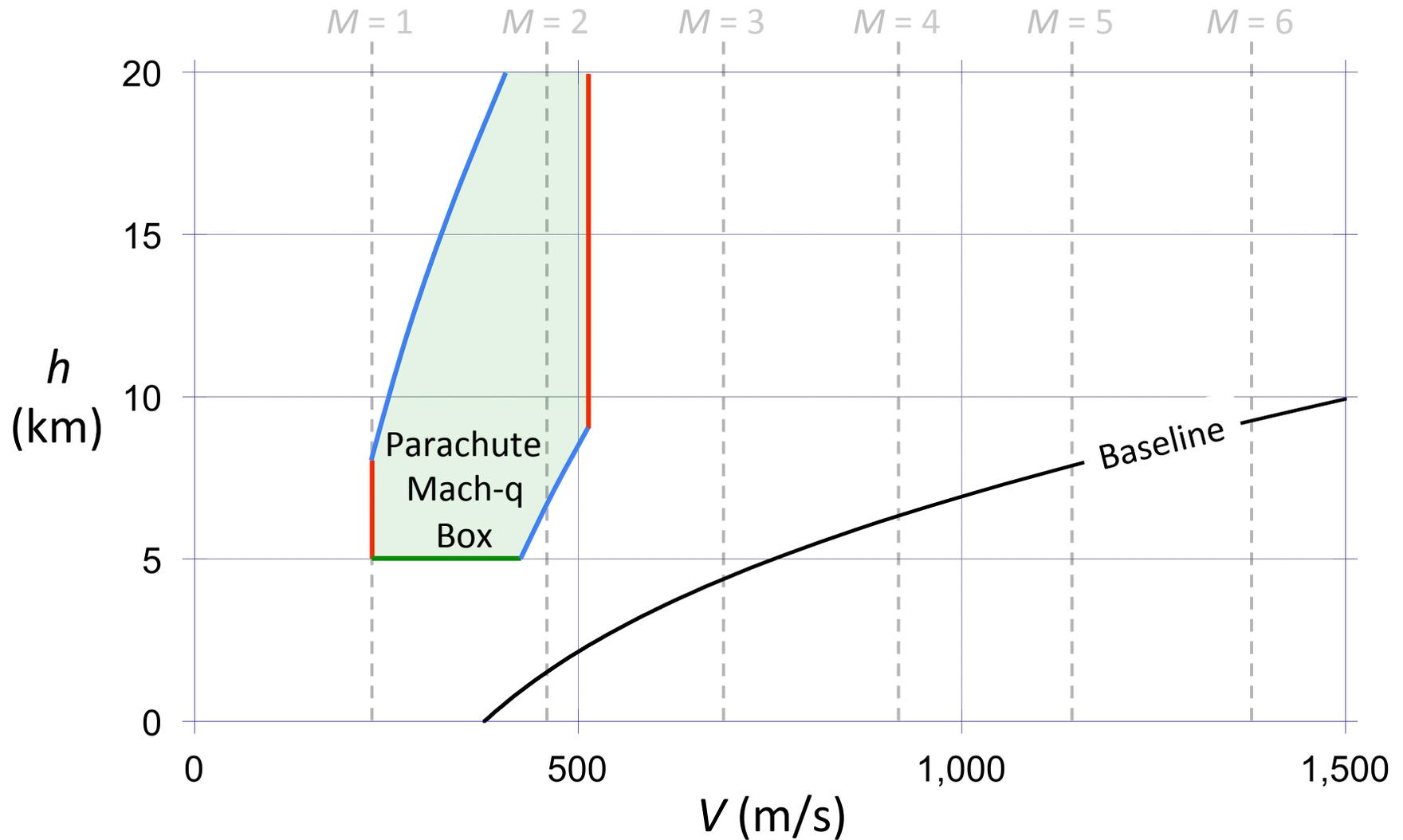
# Effect of Planet's Atmosphere and Gravity



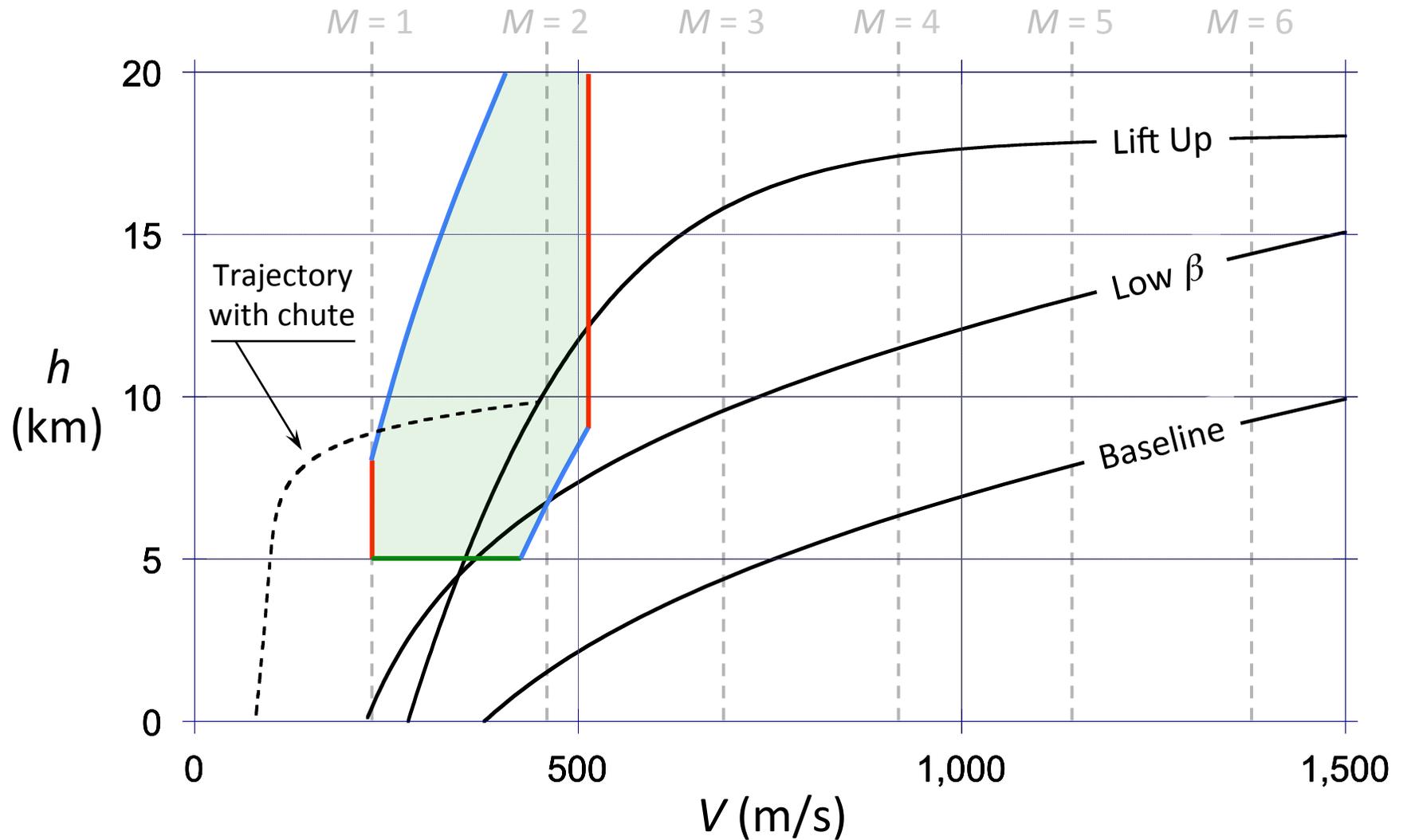
# Descent: Parachutes, IADs, and the Mach-q Box



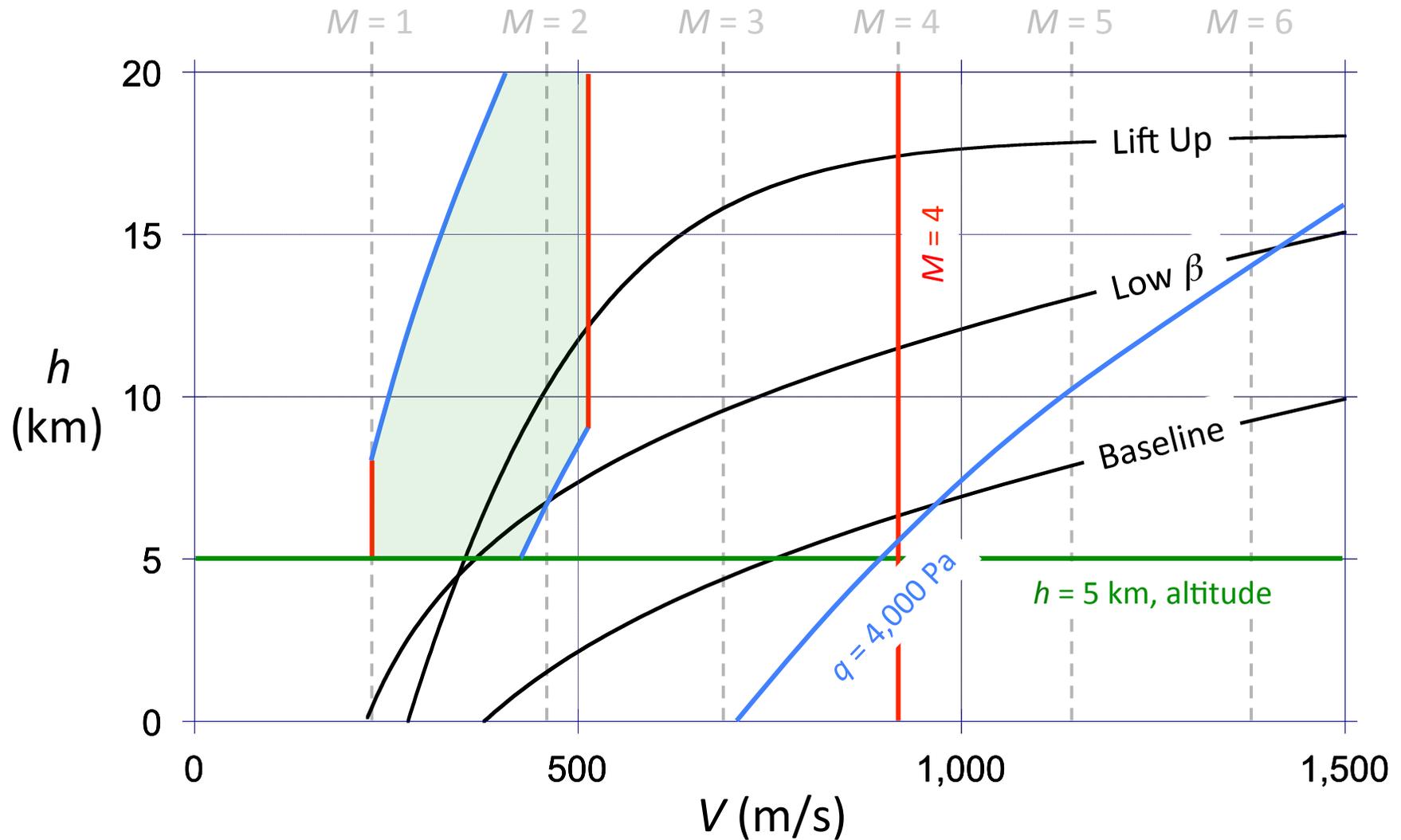
# Descent: Parachutes, IADs, and the Mach-q Box



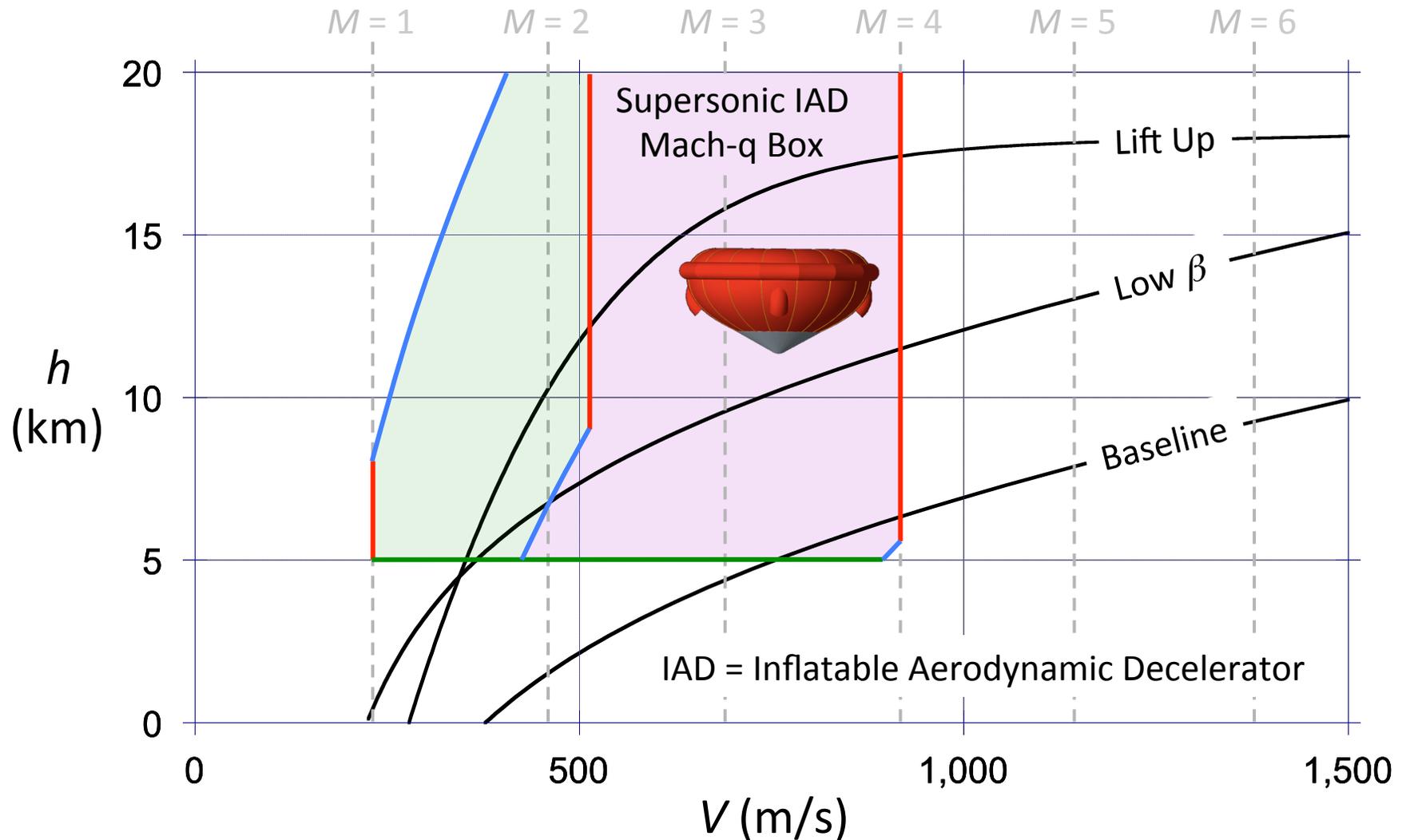
# Descent: Parachutes, IADs, and the Mach-q Box



# Descent: Parachutes, IADs, and the Mach-q Box



# Descent: Parachutes, IADs, and the Mach-q Box



# Other Considerations in Trajectory Design

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- Constraints imposed by launch vehicle – aeroshell diameter, mass
- Available entry, descent, and landing technologies and their constraints
  - aeroshell shape & performance (e.g.,  $C_D$ ,  $L/D$ )
  - thermal protection system performance (e.g. peak heat load)
  - deployable aerodynamic decelerators (parachutes, IADs)
- Timeline and altitude – how much time and altitude are needed to complete all entry, descent, and landing tasks with margin and in the presence of uncertainties?
  - deployable decelerator deployment
  - heatshield release
  - radar lock
  - propulsive descent
- Trajectory guidance
- Propulsive terminal descent
- Precision and hazard avoidance landing

# Additional Reading

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- 1) Braun, R. D. and Manning, R. M.: Mars Exploration Entry, Descent, and Landing Challenges, *Journal of Spacecraft and Rockets*, Vol. 44, No. 2, pp. 310-323, March-April 2007.
- 2) Regan, F. J. and Anandakrishnan, S. M.: *Dynamics of Atmospheric Re-Entry*, AIAA, Washington, DC, 1992. (Chapter 7: *Re-Entry Vehicle Particle Mechanics*, covers much of the material in this presentation. Of particular interest are the closed-form solutions which yield significant insights regarding the nature of trajectories. See, for example, Case 4 starting on page 195.)
- 3) Gallais, P.: *Atmospheric Re-Entry Vehicle Mechanics*, Springer-Verlag, Berlin, 2007. (Chapter 9: *Zero Angle of Attack Reentry* covers some of the material in this presentation.)
- 4) Tewari, A.: *Atmospheric and Space Flight Dynamics, Modeling and Simulation with MATLAB® and Simulink®*, Birkhäuser, Boston, 2007.
- 5) Ball, A. J., Garry, J. R. C., Lorenz, R. D., and Kerzhanovich, V. V.: *Planetary Landers and Entry Probes*, Cambridge University Press, Cambridge, UK, 2007.

# Symbols I

---

$A$	aerodynamic reference area (entry vehicle frontal area)
$a_0$	atmospheric speed of sound
$C_D$	drag coefficient
$D$	drag
$E$	total energy metric (nondimensionalized wrt. the total energy of the <i>Baseline Mars Entry</i> at atmospheric interphase)
$g$	acceleration of gravity
$g_E$	Earth acceleration of gravity
$h$	altitude above planet surface
$h_i$	altitude at initial contact with planet atmosphere (i.e., at atmospheric interphase)
$H$	atmospheric density scale height
$KE$	kinetic energy

# Symbols II

---

$K_{planet}$	convective aeroheating constant to adjust for planet's atmosphere
$L$	lift
$L/D$	lift to drag ratio
$m$	mass
$M$	Mach number
$n$	sensed load factor
$PE$	potential energy
$Q$	aerothermodynamic convective heat load metric (nondimensionalized wrt. the convective heat load of the <i>Baseline Mars Entry</i> at landing)
$\dot{Q}$	aerothermodynamic convective heat rate metric (nondimensionalized wrt. the maximum convective heat rate of the <i>Baseline Mars Entry</i> )
$q$	dynamic pressure

# Symbols III

---

$R$	planet atmosphere gas constant
$r_p$	planet radius
$T_0$	atmospheric temperature
$t$	time
$t_i$	time at initial contact with planet atmosphere (i.e., at atmospheric interphase)
$t_{landing}$	time at landing
$V$	velocity (airspeed)
$V_i$	velocity (airspeed) at initial contact with planet atmosphere (i.e., at atmospheric interphase)
$\alpha_T$	total angle of attack
$\beta$	ballistic coefficient
$\gamma$	atmosphere ratio of specific heats
$\gamma_{FPA}$	flight path angle (positive above horizon)

# Symbols IV

---

$\gamma_{FPA,i}$  flight path angle (positive above horizon) at initial contact with planet atmosphere (i.e. at atmospheric interphase)

$\rho$  atmospheric density

$\rho_0$  atmospheric density at planet surface

IAD Inflatable Aerodynamic Decelerator

# Points of Contact

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# Equations of Motion

---

$$\frac{dV}{dt} = -\left(\frac{\rho V^2}{2\beta} + g \sin \gamma_{FPA}\right)$$

$$\frac{d\gamma_{FPA}}{dt} = \frac{1}{V} \left[ \frac{V^2 \cos \gamma_{FPA}}{r_p + h} + \frac{\rho V^2}{2\beta} \left(\frac{L}{D}\right) - g \cos \gamma_{FPA} \right]$$

$$\frac{dh}{dt} = V \sin \gamma_{FPA}$$

where,

$$\beta = \frac{m}{C_D A}$$

## Assumptions

Constant acceleration of gravity

Circular planet

Equations neglect planet rotation

# Exponential Atmosphere Model

---

Examples assume exponential (isothermal) atmosphere model

$$\rho = \rho_0 \exp\left(-\frac{h}{H}\right)$$

where,

$$H = \frac{RT_0}{g}$$

Speed of sound is calculated from

$$a_0 = \sqrt{gRT_0}$$

Note that the speed of sound is constant in this simplified atmosphere model.

# Other Equations and Assumptions

---

$$M = \frac{V}{a_0}$$

$$q = \frac{1}{2} \rho V^2$$

$$n = \frac{q C_D A}{m g_E} \left[ 1 + \left( \frac{L}{D} \right)^2 \right]$$

$$KE = \frac{1}{2} m V^2$$

$$PE = mgh$$

$$E = \frac{KE + PE}{(KE + PE)_{\text{Mars Baseline @ Entry } (t_i, h_i, V_i)}}$$

# Other Equations and Assumptions

---

Nondimensional convective heat rate metric

$$\dot{Q} = \frac{K_{planet} \left( \rho^{\frac{1}{2}} V^3 \right)}{\text{Max} \left( \rho^{\frac{1}{2}} V^3 \right)_{\text{Mars Baseline}}}$$

Nondimensional convective heat load metric

$$Q = \frac{K_{planet} \left( \int_{t_i}^t \rho^{\frac{1}{2}} V^3 dt \right)}{\left( \int_{t_i}^{t_{landing}} \rho^{\frac{1}{2}} V^3 dt \right)_{\text{Mars Baseline}}}$$

$K_{planet} = 1$  for Mars;  $K_{planet} = 1.092$  for Earth

Same aeroshell nose radius assumed for all cases

# Example Cases

	Mars Baseline	Mars Aerocapture	Mars Low $\beta$	Mars High $\beta$	Mars Lift Up	Mars Lift Down	Earth
<b>Vehicle</b>							
$\beta$ (kg/m <sup>3</sup> )	125	125	<b>75</b>	<b>175</b>	125	125	125
$L/D$	0.00	0.00	0.00	0.00	<b>0.15</b>	<b>-0.15</b>	0.00
$m$ (kg)	1,000	1,000	1,000	1,000	1,000	1,000	1,000
$A$ (m <sup>2</sup> )	5.333	5.333	<b>8.889</b>	<b>3.810</b>	5.333	5.333	5.333
$C_D$	1.5	1.5	1.5	1.5	1.5	1.5	1.5
<b>Atmosphere</b>							
$T_0$ (K)	210	210	210	210	210	210	<b>288</b>
$\gamma$	1.33	1.33	1.33	1.33	1.33	1.33	<b>1.40</b>
$R$ (J/kg•K)	188.9	188.9	188.9	188.9	188.9	188.9	<b>287.1</b>
$H$ (km)	10.607	10.607	10.607	10.607	10.607	10.607	<b>8.429</b>
$\rho_0$ (kg/m <sup>3</sup> )	0.0160	0.0160	0.0160	0.0160	0.0160	0.0160	<b>1.225</b>
$a_0$ (m/s)	229.7	229.7	229.7	229.7	229.7	229.7	<b>340.2</b>
<b>Geodesy</b>							
$r_p$ (km)	3,396	3,396	3,396	3,396	3,396	3,396	<b>6,378</b>
$g$ (m/s <sup>2</sup> )	3.74	3.74	3.74	3.74	3.74	3.74	<b>9.81</b>
<b>Initial Conditions</b>							
$h_i$ (km)	125	125	125	125	125	125	125
$V_i$ (m/s)	6.0	6.0	6.0	6.0	6.0	6.0	6.0
$\gamma_{FPA,i}$ (deg)	-15.0	<b>-9.4</b>	-15.0	-15.0	-15.0	-15.0	-15.0

Values in **bold red** are changes from the *Baseline Mars Entry* example.