Tutorial on Ablative TPS Engineering Methods

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Outline

• Engineering methods - why?
• Convective heating in air and CO₂
  – Laminar boundary layer
  – Turbulent boundary layer
• Boundary layer transition
• Radiative heating in air and CO₂-N₂
• Heat load scaling relations
  – Ballistic coefficient
  – Flight path angle
• Reference list
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Rationale

With present computational abilities, why use engineering methods?

- Some applications of simple relationships for calculating non-ablating convective and radiative heating
  - Negligible computation time
  - Included in most atmospheric trajectory codes-stag. pt. heating
  - Initial estimates of heating rates and loads for use during conceptual design stage
  - Categorizing the type of TPS material required
  - Comparing with values from CFD-type design codes
  - Estimating margins in final design
Convective Heat Transfer - 1

Definitions:

\[ St = \frac{\dot{q}_w}{\rho_e u_e (h_{aw} - h_w)} \]
\[ Pr = \frac{c_p u}{k} \]
\[ Nu = \frac{\dot{q}_w}{k_w (h_{aw} - h_w)} \]

Modified Reynolds analogy:

\[ St = \frac{c_f}{2} Pr^{-2/3} \]
\[ \text{but } c_f \sim \text{Re}_w^{-n} \quad \text{(lam b.l. } n = 0.5, \text{ turb b.l. } n = 0.8) \]

from

\[ \frac{Nu}{Re} = (St)(Pr), \quad \frac{Nu}{Re} \sim c_f Pr^{1/3} \sim \text{Re}^{-n} \text{Pr}^{1/3} \Rightarrow \frac{Nu}{Re^{1-n}} \sim \text{Pr}^{1/3} \]

Therefore, lam b.l. parameter is \( \frac{Nu}{Re^{0.5}} \) and turb is \( \frac{Nu}{Re^{0.2}} \)

Geometric relations:

\[ \frac{\dot{q}_c}{\dot{q}_{fp}} = \left( \frac{2-n}{1-n} \right)^n = 1.732 \text{ (lam)} \text{ or } 1.176 \text{ (turb)} \]

In high speed flows, heating depends on
- Conduction through the b.l.
- Heat of recombination from species diffusing through the b.l. to the surface
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Convective Heat Transfer - 2

For the laminar boundary layer at high speeds (Ref. 1)

\[ \frac{Nu}{Re^{0.5}} = C_f \frac{T_W^b}{V_x^a} \text{ where } a = 0.1 - 0.2 \text{ and } b = 0.3 - 0.4 \]

\( C_f \) depends on body geometry: stagnation point, cone, flat plate
For a laminar boundary layer in air, for the stagnation point.
Using \( a=0.3575 \) and \( b=0.145 \), we get (Ref. 1)

\[ \dot{q}_{w0} = 1.83 \times 10^{-4} \left( \frac{\rho_x}{r_n} \right)^{0.5} V_x^3 \left( 1 - \frac{h_w}{h_t} \right) \text{ W/m}^2 \]

And for an effectively sharp cone of \( \delta_c \geq 15^\circ \)

\[ \dot{q}_{wc} = 4.03 \times 10^{-5} \left( \frac{\rho_x \cos \delta_c}{x} \right)^{0.5} V_x^{3.2} \sin \delta_c \left( 1 - \frac{h_w}{h_{aw}} \right) \text{ W/m}^2 \]

where \( h_{aw} = h_x + 0.5(Pr)^{0.5} V_x^2 \)
Convective Heat Transfer - 3

- For a CO₂ atmosphere (e.g., Mars and Venus), for the laminar boundary layer stagnation point (Ref. 2)

\[ q_{w0} = 1.35 \times 10^{-4} \left( \frac{\rho_\infty}{\rho_n} \right)^{0.5} \frac{V_\infty^{3.04}}{h_t} \left( 1 - \frac{h_w}{h_t} \right) \text{ W/m}^2 \]

- For the turbulent boundary layer, see Ref. 3
Comparison of Laminar Convective Heat Transfer Parameter Calculations

Space Technology Division

![Graph comparing Nl_w/Re_w vs Flight Velocity, km/sec for various researchers and geometries.](image)
Stagnation point convective heating comparisons

(REF. TP 2914)
Convective heating data

Space Technology Division
Total thermal conductivity comparison

Ref. 5

P = 1 atm
Stanton number distribution

Ref. 1

Space Technology Division
Comparison of CL heating calculations with STS-1 Flight Data

Ref. 1
Convective heating correlations in CO$_2$

data from various sources are plotted against ($h_g - h_p$). For air, the shaded area represents the majority of shock-tube data from a large number of sources. The solid lines represent equation (11) with the appropriate values of $C$ and $N$. The data and equation (11) agree well for all the gases. Also, equation (11) agrees very well with the theory of reference 4 for air and CO$_2$ and reference 20 for air.

(REF. 6 & 7)
Pioneer Venus Sounder (Large) Probe
Stagnation Point Results (Refs. 8 & 9)

Convective Heating
Radiative Heating

- Design Data: scanned from PV CDR report
- Traj Results: 3DoF reconstruction of design trajectory
- DPLR Results: fully catalytic ("error" bars show modeling variability)
- NEQAIR Results: tangent slab corr. = 0.75; non-adiabatic corr. = 0.75

95% of radiation from CO(4+).
Pioneer Venus Sounder (Large) Probe
Flank Results

Engineering calculation method:

- Flat plate turbulent b.l. in air (Ref. 1) with $h_w$ for CO$_2$ (Ref. 10)
- Geometric transformation to cone - multiply by 1.176
- From Ref. 3, for $V_\infty = 6000$-10000 m/s, use

\[
\left( \frac{\dot{q}_{CO_2}}{\dot{q}_{air}} \right)_{turb} = 271V_\infty^{-0.59}
\]

- Design data: scanned from PV CDR report
- DPLR results: fully catalytic, Baldwin-Lomax (error bars show modeling variability)
Boundary Layer Transition

- Determination of when in trajectory and where on body boundary layer transition occurs is mostly empirical
- Ground facility tests
  - Ablating material facilities (arc jets)
    - Very low Reynolds no., laminar boundary layer only
  - High-speed wind tunnels - ineffective because of disturbances
    - Pressure fluctuations in stream
    - Turbulent boundary layer on wall disturbs flow result in early transition
- High-speed flight data best by far, but sparse


**Approach**

Transition caused by changes in the B.L. velocity and temperature profiles (Refs. 11-14)

1) Amplification of disturbances in B.L.  
   B.L. edge Re based on length, $Re_L$, or momentum thickness, $Re_\theta$  
   $Re_\theta < 200$ for approx. $M < 1.5$ (STS, etc.)

2) Surface roughness max. height, $k$  
   $Re_k < 100$ (STS, USAF-PANT, etc.)

3) Ablation mass-injection (Apollo)

4) Combined mass-injection and roughness (USAF-PANT)

5) Cross-flow caused by $\alpha \neq 0$, neglect if $\alpha << \theta_c$  
   Heavy reliance on CFD and material response codes (GIANTS, DPLR and FIAT) for inputs
B.L. Transition Correlations

Ref 13

Figure 12 - Comparison of Smooth and Distributed Roughness Transition Results.

ML-20
MER-B Undershoot GASP CFD
TCM5, Gamma=-12.25 deg

Transition Criteria

Laminar/Turbulent Comparison

Ref. 15

ML-21
Radiative Heating

For Earth and Mars, for stagnation point (see Ref. 16)
\[ \dot{q}_r = C_i r_n^a \rho_\infty^b f_i(V_\infty) \text{ where sub } i = E \text{ or } M \]

For Earth:
\[ a = f(\rho_\infty, V_\infty), \quad b = 1.22, \quad C_E = 4.736 \times 10^4 \]

For Mars:
\[ a = 0.526, \quad b = 1.19, \quad C_M = 2.35 \times 10^4 \]
\[ f_E(V_\infty) \text{ and } f_M(V_\infty) \text{ are tabulated} \]

(Note ranges of applicability of \( r_n \) and \( \rho_\infty \))

For Venus - steep entry angles, high stagnation pressure, see Ref. 17 for tabulated values (90% CO\(_2\) - 10% N\(_2\) atmosphere)
(actual atmospheric composition is 96.5% CO\(_2\) - 3.5% N\(_2\))
\[ b = 1.2 \]
Comparison of calculated heating with Fire II Data

\[ V_e = 11.37 \text{ km/s (Refs. 18-21)} \]
Convective (Case 3) and Radiative Heat Fluxes (Ref. 22)

\[ V_\infty = 13,280 \text{ m/s}, \quad \rho_\infty = 1.078 \times 10^{-4} \text{ kg/m}^3, \quad \text{alt} = 65 \text{ km}, \quad p_s = 0.18 \text{ atm} \]
# Stagnation Point Heating Rate Comparisons

AIR (Ref. 23)

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ML-25
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95% of radiation from CO(4+)
Mars Pathfinder Growth History

Space Technology Division

![Graph showing the growth history of Mars Pathfinder, with data points for mass and ballistic coefficient from 1992 to 1996.](Image)
Heating Pulse Parameters

Aerodynamic heating rate (non-ablating)

\[ \frac{dq}{dt} = C_f(L) \rho^n V^m \left(1 - \frac{h_w}{h_r}\right) \sim \rho^n V^m \]

(assuming \( \frac{h_w}{h_r} << 1 \), and \( C, f(L) \) are independent of \( \rho \) and \( V \))

Heat load \( q \sim \int \rho^n V^m dt \)

During heating pulse \( \frac{W \sin \gamma}{D} = \frac{mg \sin \gamma}{\frac{1}{2} \rho V^2 C_D A} \ll 1 \)

Thus \( -\frac{dV}{dt} = \frac{C_D A}{m} \rho V^2 \) or \( dt = -2 \frac{m}{C_D A \rho V^2} dV \)

Assuming that the ballistic coefficient, \( \frac{m}{C_D A} \), and flight path angle, \( \gamma \), are constant, permits relating \( \rho \) and \( V \) (the Allen-Eggers equation*) and integration of the heat load expression.

(see Tauber, J. Spacecraft & Rockets, July 1970, Ref. 24)

*For \( \frac{L}{D} = 0 \)
Entry Heat Load Dependence

Heat Load: 

\[ q \sim \left( \frac{m}{C_D A} \right)^n (\sin \gamma)^{n-1} \]

For laminar boundary layer: \( n = 0.5 \)
For turbulent boundary layer: \( n = 0.8 \)
For shock-layer radiation (depends on atmospheric composition, shock layer thickness, etc.)

- Mars atmosphere: \( n = 1.19 \)
- Earth atmosphere: \( n = 1.22 \)
- Venus atmosphere: \( n = 1.20 \)
- Jupiter atmosphere: \( n = 1.17-1.45 \)

(Note strong dependence of radiative heat load on \( \frac{m}{C_D A} \))

Digress: What fraction of entry KE must be dissipated in form of heat by vehicles?
Examples: Jupiter Galileo probe (ablating) \( \approx 0.1\% \)
Shuttle orbiter (radiation cooling) \( \approx 0.5\% \)
Entry Heat Load Variation with Flight Path Angle
(Relative Values)

Relative heat load, non-ablating

Relative flight path angle (deg)

Radiative
- Jupiter
- Earth
- Venus

Convective
- Turbulent
- Laminar
Summary

- Engineering heating equations are useful and adequately accurate for many intended applications
  - Comparisons in air for generic sample return missions ($V_E = 11-14$ km/s) and for Pioneer-Venus large probe, show stagnation point convective heating 10-20% below CFD
  - Radiative heating 10-30% above NEQAIR
  - But, sum of convective and radiative heating is 10% higher than measured peak Fire II flight rate
  - Turbulent convective heating is overpredicted by up to 30% in air and primarily CO$_2$ atmospheres
Summary (concluded)

• Boundary layer transition
  – Ground facilities predict earlier transition than flight
  – Use up to 5 empirical criteria

• Scaling of heat load with ballistic coefficient (especially) and entry angle depends on dominant heating mechanism: laminar or turbulent boundary layer or radiation
Heating Pulse Parameters

Aerodynamic heating rate (non-ablating)
\[
\frac{dq}{dt} = Cf(L)\rho^n V^m \left(1 - \frac{h_w}{h_r}\right) \sim \rho^n V^m
\]
(assuming \(\frac{h_w}{h_r} << 1\), and \(C, f(L), n\), and \(m\) are independent of \(\rho\) and \(V\))

Heat load \(q \sim \int \rho^n V^m dt\)

During heating pulse
\[
\frac{W \sin \gamma}{D} = \frac{mg \sin \gamma}{\frac{1}{2} \rho V^2 C_DA} \ll 1
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Thus
\[
-\frac{dV}{dt} = \frac{1}{2} \frac{C_DA}{m} \rho V^2 \text{ or } dt = -2 \frac{m}{C_DA} \rho V^2 dt
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ML-33
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