magnetic induction at Europa: the Clipper view

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introduction

- Europa moves through Jupiter's magnetic field
- the magnetic field imposed upon Europa
  will induce a field within Europa
- the strength of the induced field
  is indicative of a conducting layer
- it measures conductance = conductivity * thickness

objective of current study

assess how well

- magnetic field measurements
  - made by the current Clipper concept
  - during 45 flyby encounters with Europa

could be used to

- constrain induction effects
  - within Europa
  - at several different forcing periods

note on magnetic field units

- cgs unit: Gauss
  Earth surface field is 0.3-0.6 Gauss

- SI unit: Tesla
  Tesla = 10^4 Gauss

- convenient unit: nano-Tesla (nT) = 10 micro-Gauss

Serbian 100 dinara note includes unit definition

outline

4 views of magnetic field

- at Jupiter
- imposed upon Europa
- induced within Europa
- at Clipper

basics of magnetic induction

The Maxwell-Faraday equation

$$\nabla \times E = -\partial B / \partial t$$

states that

a time-varying magnetic field ***B*** , is always associated with

a spatially-varying electric field ***E***, and vice-versa.

In particular,

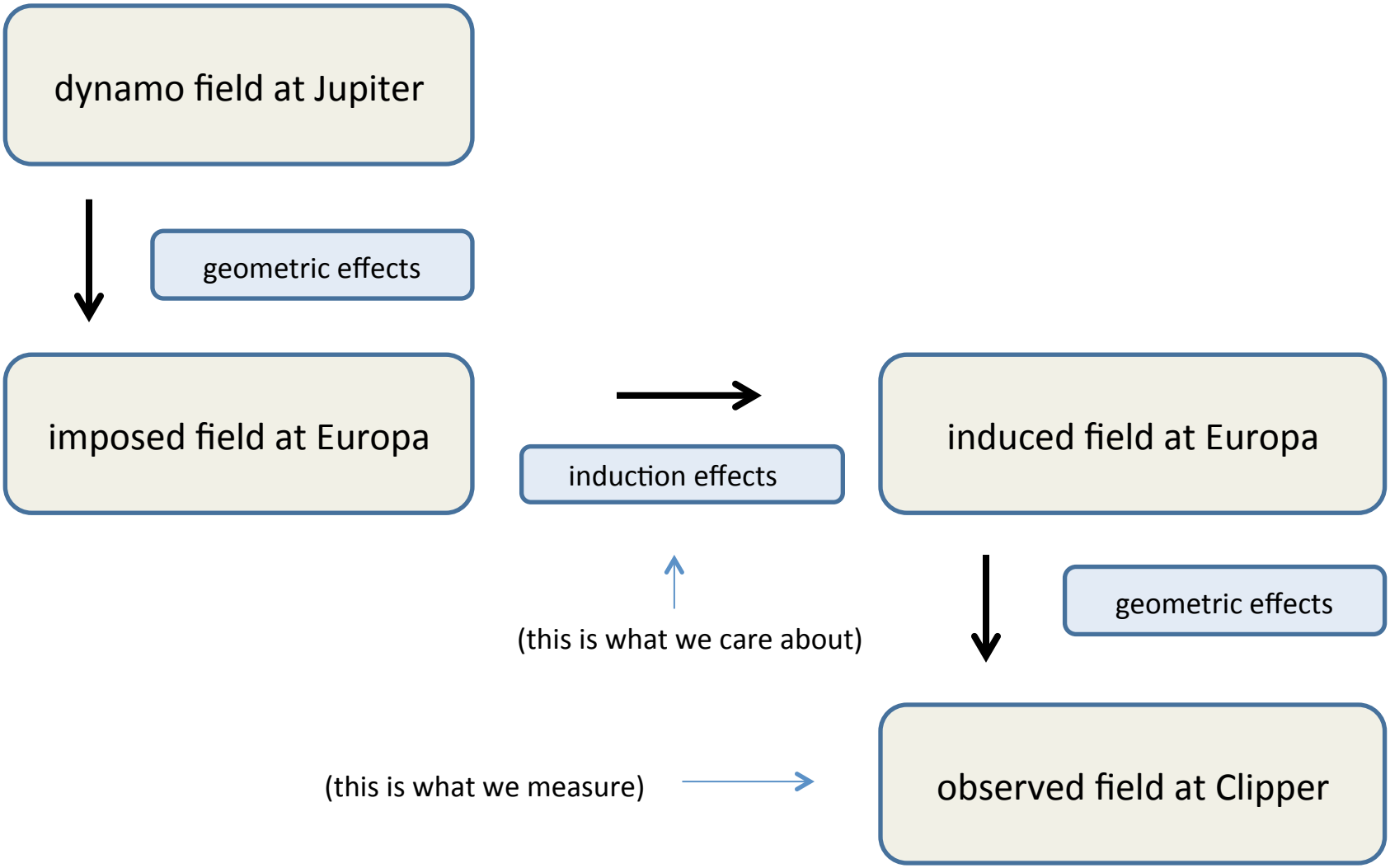
1) a time-varying magnetic field, when imposed upon an electrically conductive body (like Europa), will induce
2) electrical currents on the surface, which attempt to oppose the imposed magnetic field. Those currents generate a magnetic field , which can be measured external to the body.

A perfect conductor (Faraday cage) will succeed in preventing the imposed field from penetrating into the interior.

A body with radial variations in electrical conductivity will generate an induced magnetic field, whose strength will depend upon

- strength and frequency of imposed field variations
- radial conductivity profile

process flow chart



magnetic field at Jupiter

field at Jupiter

At a position, with spherical coordinates (r, θ, φ), relative to Jupiter

- magnetic scalar potential

$$V[r,\theta,\varphi] = R \sum_{n=1} \left(\frac{R}{r}\right)^{n+1} \sum_{m=0}^{n} P_{n,m}[\sin[\theta]] \left(g_{n,m} \cos[m\,\varphi] + h_{n,m} \sin[m\,\varphi]\right)$$

- magnetic vector force field

local current contribution

$$B = -\nabla V + b$$

$$B_r = \frac{\partial V}{\partial r}; \quad B_\theta = \frac{1}{r}\frac{\partial V}{\partial \theta}; \quad B_\varphi = \frac{1}{r\,cos[\theta]}\frac{\partial V}{\partial \varphi}$$

(spherical version)

$$B_x = \frac{\partial V}{\partial x}; \quad B_y = \frac{\partial V}{\partial y}; \quad B_z = \frac{\partial V}{\partial z}$$

(Cartesian version)

field at Jupiter

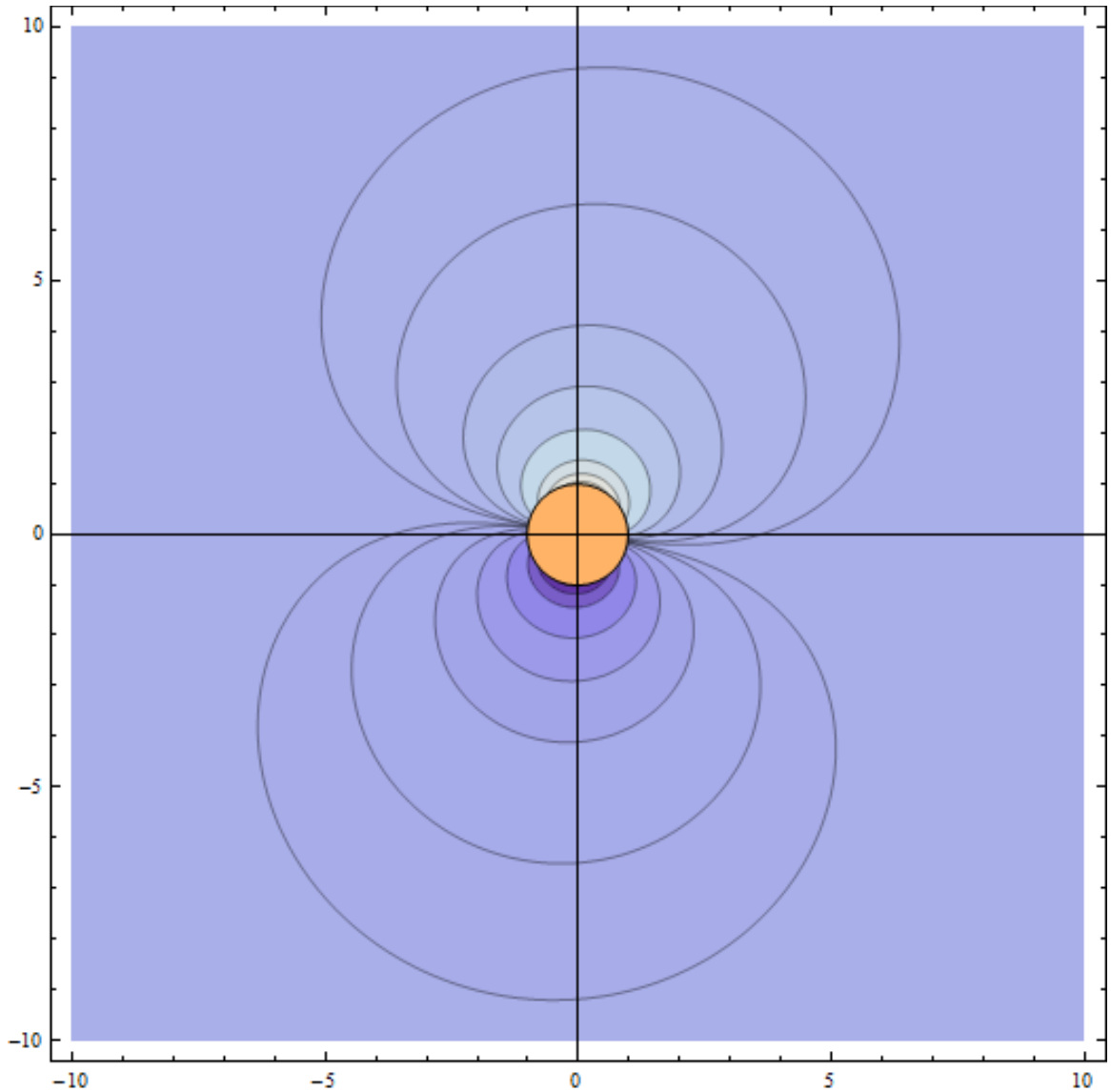
Connerney et al. (1998) JGR, 103, 11,929-11,9393

n	m	g[n,m]	h[n,m]
1	0	+4.242	
1	1	-0.659	+0.241
2	0	-0.022	
2	1	-0.711	+0.487
2	2	-0.403	+0.072
3	0	+0.075	
3	1	-0.155	+0.198
3	2	-0.180	-0.388
3	3	+0.342	-0.224

← tilted dipole

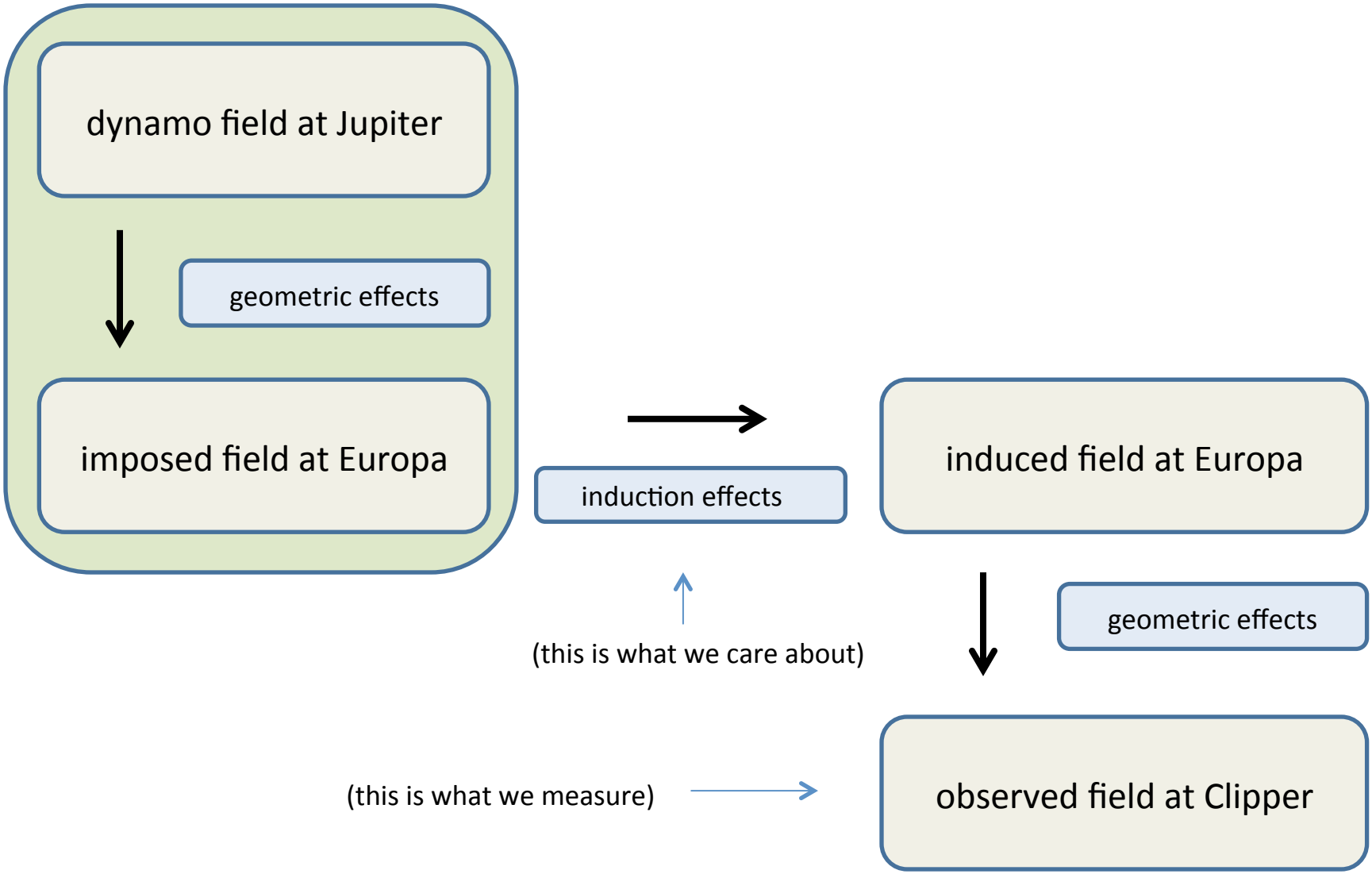
R = 71,398 km

field at Jupiter



magnetic field imposed upon Europa

Jupiter field at Europa



Jupiter field at Europa

time variations from two sources

- rotation of Jupiter
  - frequency: $\omega = 2\pi/9.925$ hr
  - inertial period: 9.925 hr
  - period as seen at Europa: 11.23 hr

- orbit of Europa
  - frequency: $n = 2\pi/85.2434$ hr
  - a = 670,900 km = 9.3966 R_{jup}
  - ecc = 0.00938
  - inc = 0.470 deg

Jupiter field at Europa

time series:

compute magnetic field vector at Europa

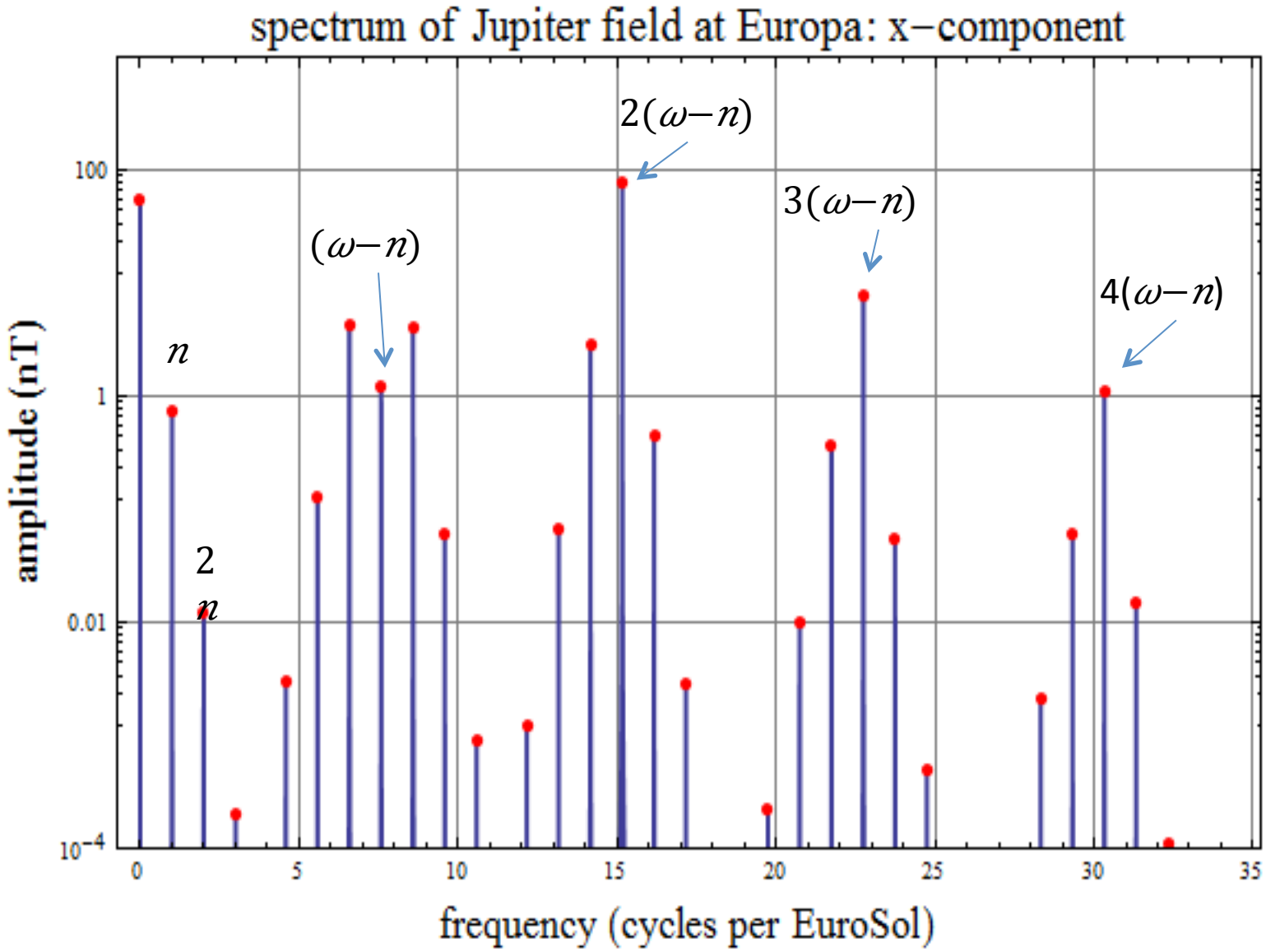
- rate: 1 sample per hour
- duration: 5 years

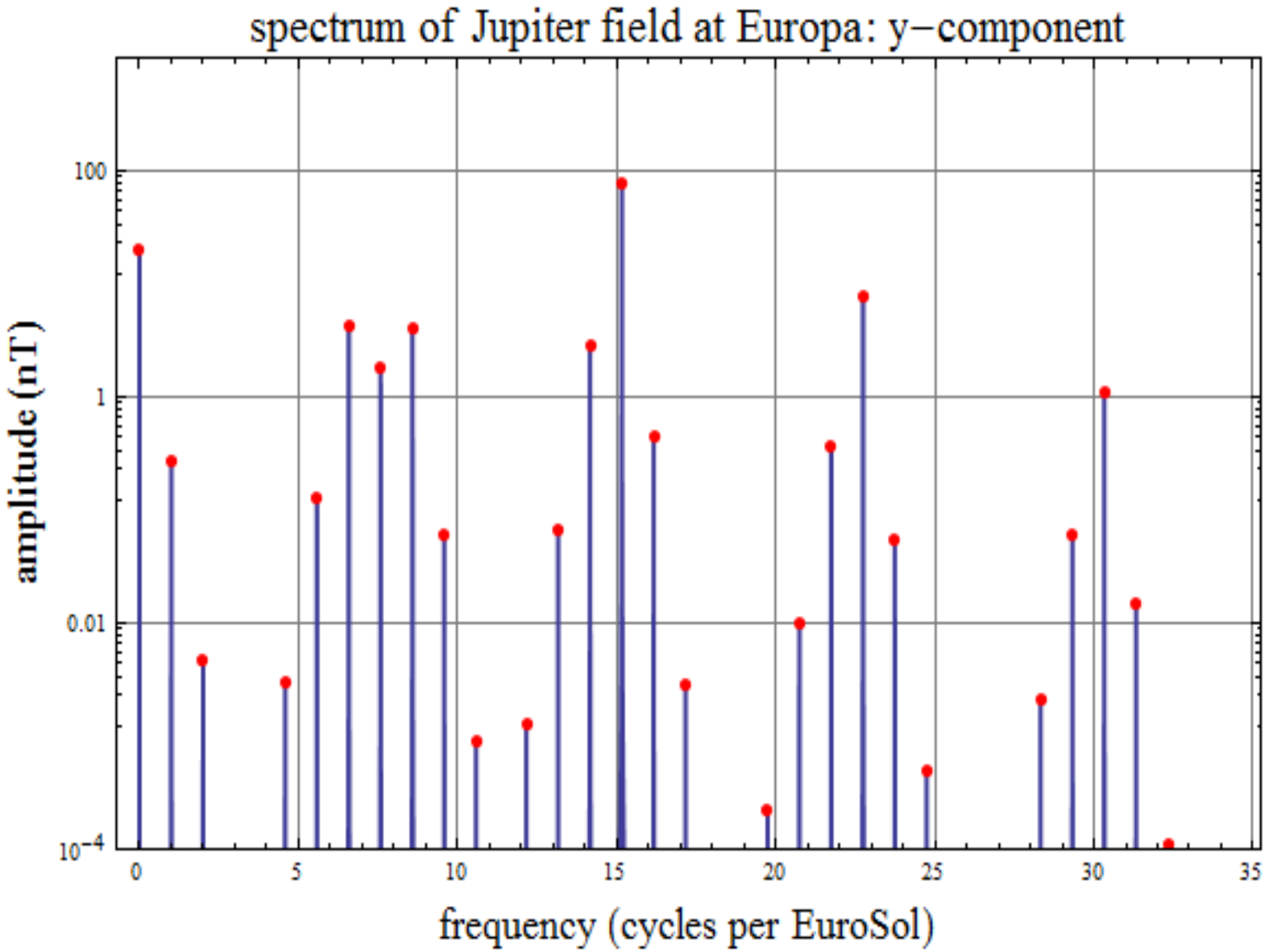
Fourier spectra:

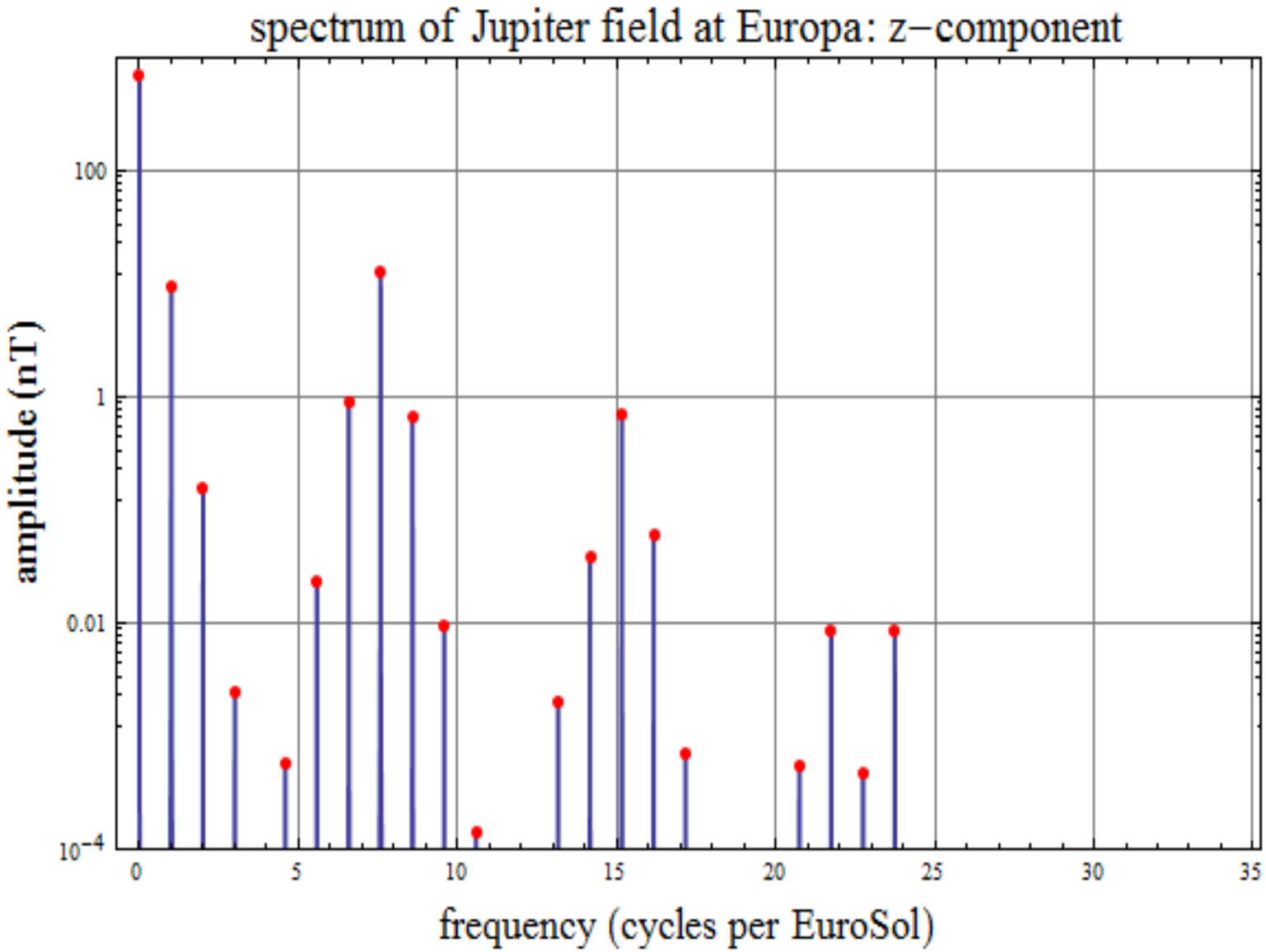
significant lines at

$f = \{1,2,3\}\, n$ (orbital period and harmonics)

$f = \Omega + \{-3,-2,-1,0,1\}\, n$ (beats between rotation and orbit)

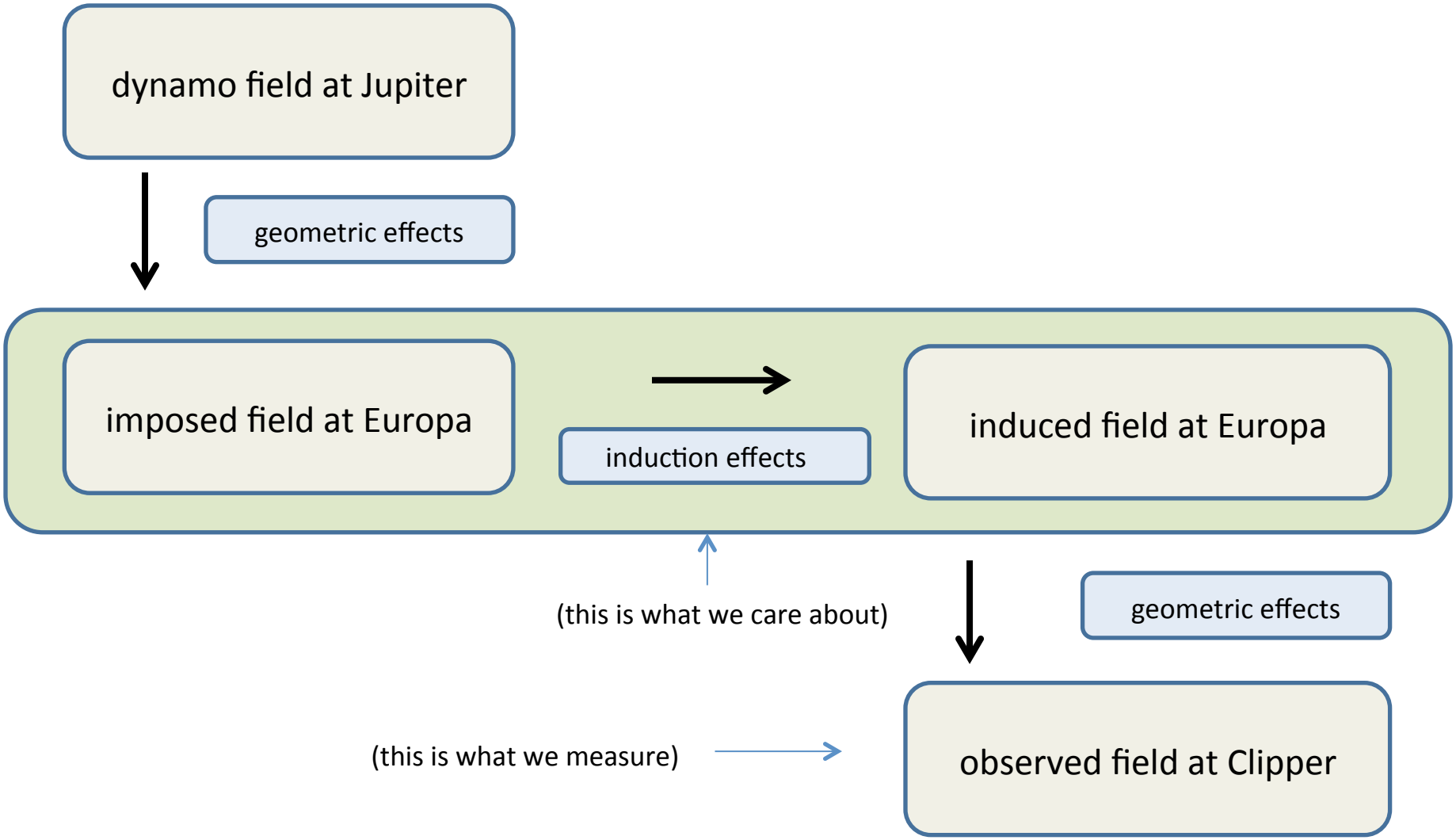

spectrum of Jupiter field at Europa: x−component
amplitude (nT)
frequency (cycles per EuroSol)
100
1
0.01
10^{-4}
0
5
10
15
20
25
30
35
n
$2n$
$(\omega-n)$
$2(\omega-n)$
$3(\omega-n)$
$4(\omega-n)$


spectrum of Jupiter field at Europa: y−component
amplitude (nT)
100
1
0.01
10⁻⁴
0
5
10
15
20
25
30
35
frequency (cycles per EuroSol)


spectrum of Jupiter field at Europa: z−component
amplitude (nT)
100
1
0.01
10⁻⁴
0
5
10
15
20
25
30
35
frequency (cycles per EuroSol)

magnetic field induced in Europa

induced field in Europa



induced field in Europa

time variations in Jupiter's magnetic field, at Europa, cause an induced magnetic field, centered on Europa, which tends to cancel the imposed field

the induced field is

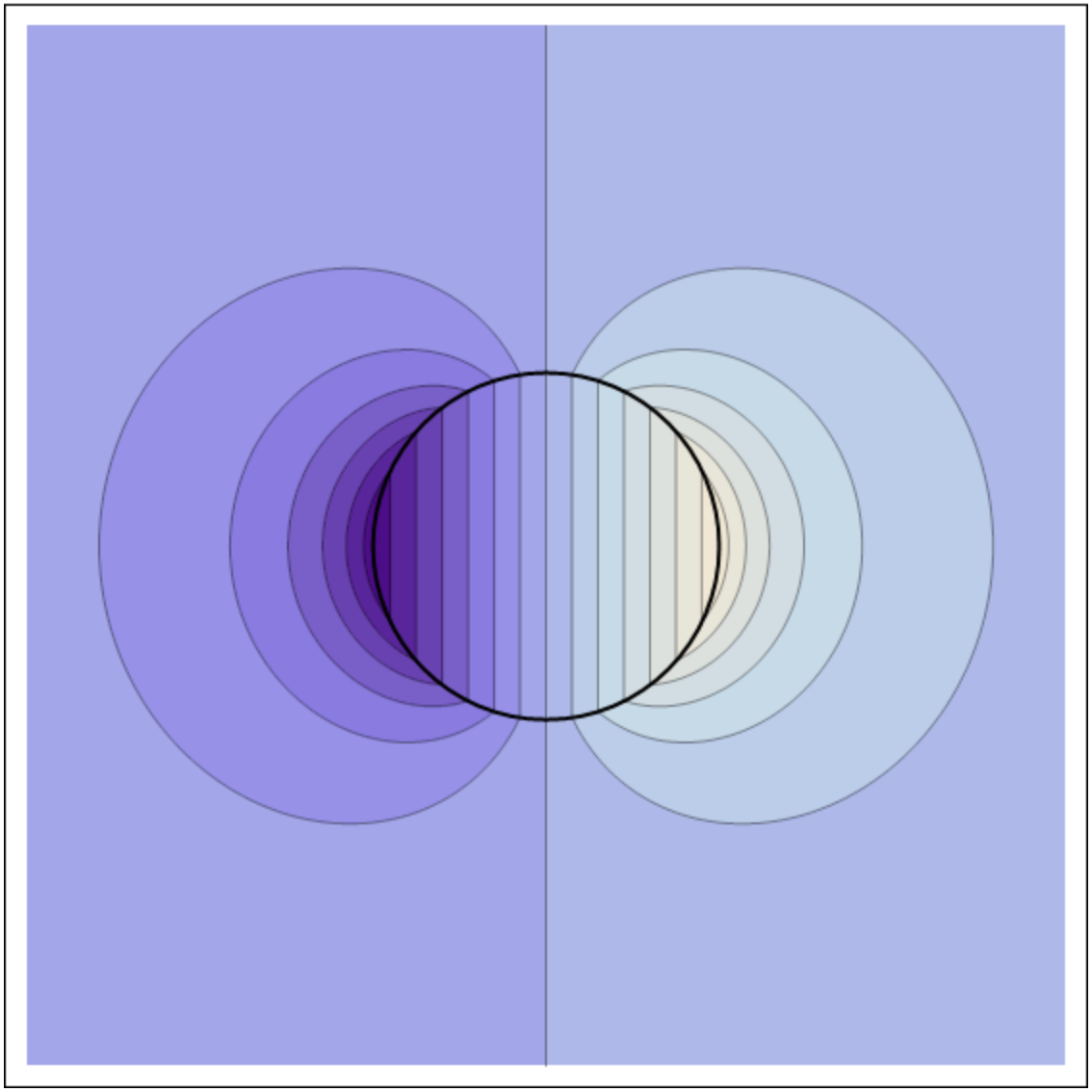
- strictly dipolar,
- anti-parallel to the imposed field, and
- has a frequency dependent amplitude

the amplitude is

- diagnostic of internal structure
- measuring it is the main point of the experiment

induced field in Europa

dipolar potential inside and outside uniform sphere



induced field in Europa

In response to a time dependent imposed magnetic field

$$B=\{B_x, B_y, B_z\}$$

a conductive spherical body of radius R will have an induced field B*, with value, at external position

$$p=\{p_x, p_y, p_z\}$$

which is given by

$$B^* [B,p]=-Q/2 \ (R/r)^3 \ (3(B\cdot p/r^2)p-B)$$

where

$$r=\sqrt{p\cdot p}$$

The scale factor *Q* will depend upon both

- conductivity within the body, and
- period of variation of the imposed field

induced field in Europa

Assume a Poisson* series expansion for the magnetic field,
due to Jupiter, at Europa:

$$B_x[t]=\sum_j B_{x,j}[t]=\sum_j C_{x,j}\cos[\sigma_j t]+S_{x,j}\sin[\sigma_j t]$$

$$B_y[t]=\sum_j B_{y,j}[t]=\sum_j C_{y,j}\cos[\sigma_j t]+S_{y,j}\sin[\sigma_j t]$$

$$B_z[t]=\sum_j B_{z,j}[t]=\sum_j C_{z,j}\cos[\sigma_j t]+S_{z,j}\sin[\sigma_j t]$$

or briefly,

$$B[t]=\sum_j B_j[t]$$

*Poisson series: similar to Fourier series (weighted sum of sines and cosines) but frequencies are not all integer multiples of a single base value.

induced field in Europa

The induced field, at external position p, is given by

$$B^{*}[t] = -1/2 \sum_j Q_j \, (R/r)^3 \, (3(B_j \cdot p/r^2)p - B_j)$$

For a solid sphere, made of a perfect conductor, $Q = 1$.

Our challenge is to infer, from measurements at Clipper,
the various Q values for Europa, at different frequencies.

magnetic induction response

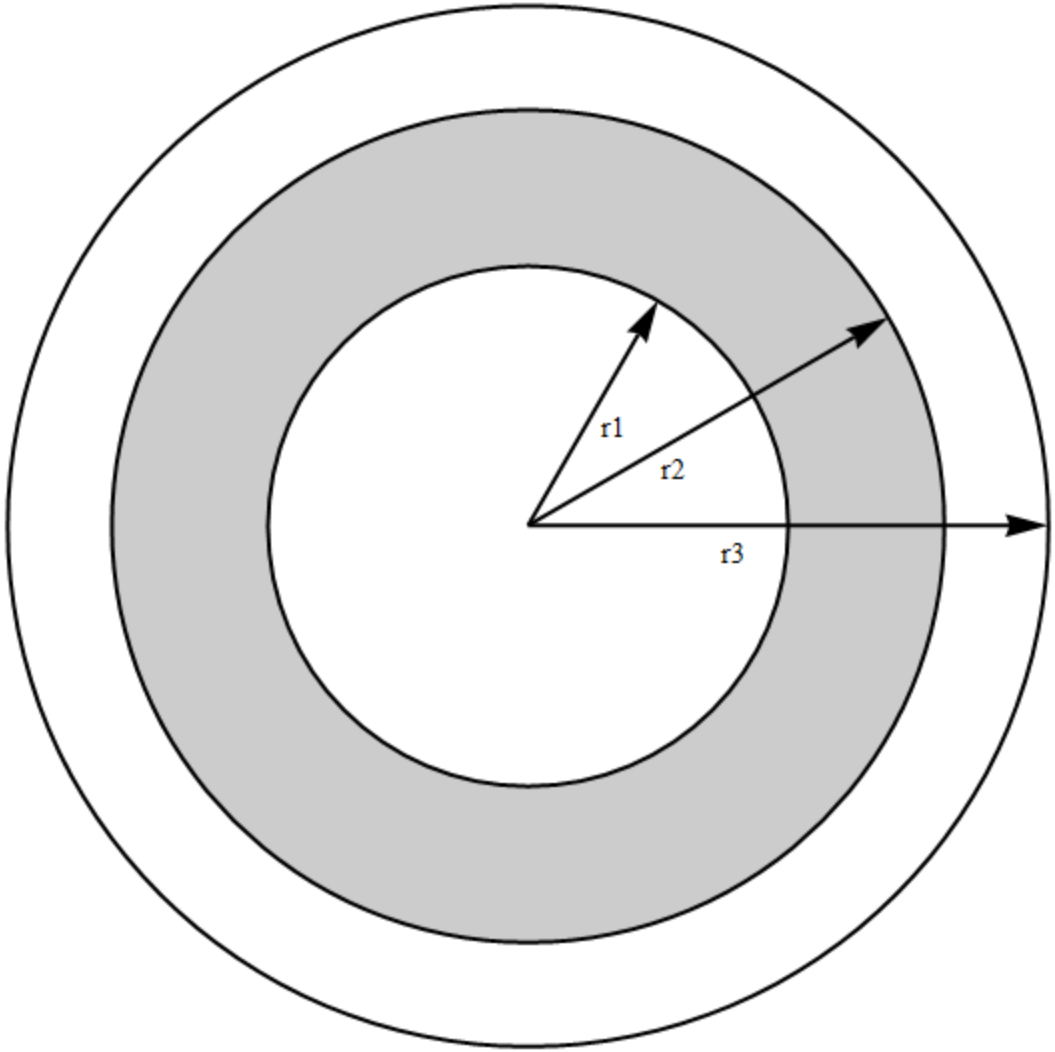
assume 3 layer model

- silicate interior
- salty ocean
- ice shell

compute induced response

magnetic induction response

3 layer model: gray represents ocean



magnetic induction response

in response to an imposed magnetic field **B**, a conductor has electric currents induced in it, which tend to diminish the magnetic field inside the conductor

need to find solutions to (diffusion equation)

$$\nabla^2 B = \mu_0 \, \sigma \, \frac{\partial B}{\partial t}$$

complex wavenumber:

$$k[\sigma, \omega] = \sqrt{-i \, \mu_0 \, \sigma \, \omega}$$

complex response function:

$$Q[r1, r2, r3,][\, \sigma, \omega] = \left(\frac{r2}{r3}\right)^3 \frac{F \;\; J[5/2, \;\; s2] \;\; - \; J[-5/2, \;\; s2]}{F \, J[1/2, \;\; s2] \;\; - \; J[-1/2, \;\; s2]}$$

where

$$F[r1, \sigma, \omega] = \frac{s1 \;\; J[-5/2, \;\; s1]}{3 \;\; J[3/2, \;\; s1] - s1 \;\; J[1/2, \;\; s1]}$$

and normalized radii are

$$s1 = \; r1 \, k[\sigma, \omega]$$

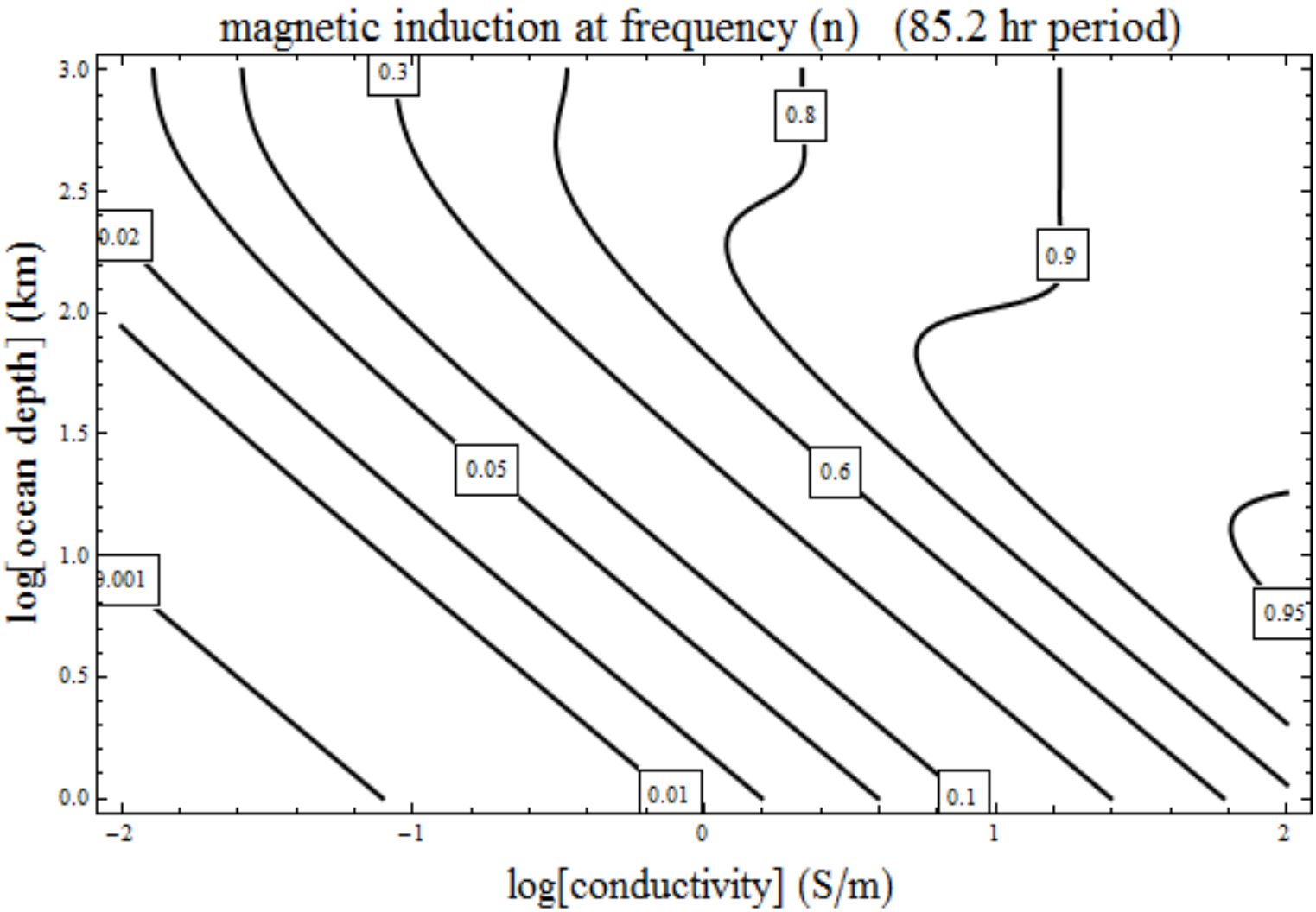
$$s2 \; = \; r2 \, k[\sigma, \omega]$$

magnetic induction response

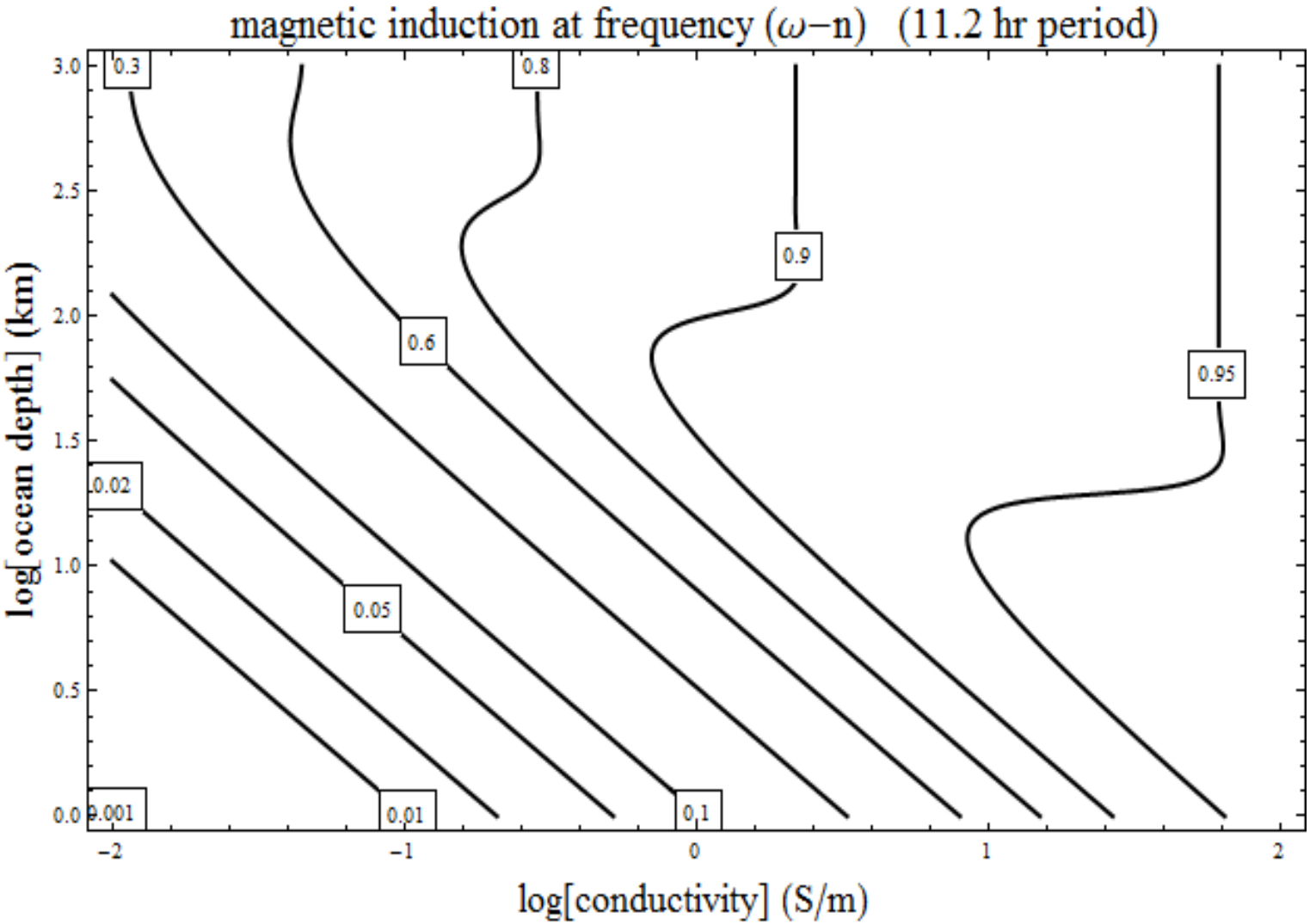
at Europa, there are two main periods of imposed magnetic field variation

period (hr)	amplitude (n T)	source
11.2	250	Jupiter rotation
85.2	15	eccentric orbit

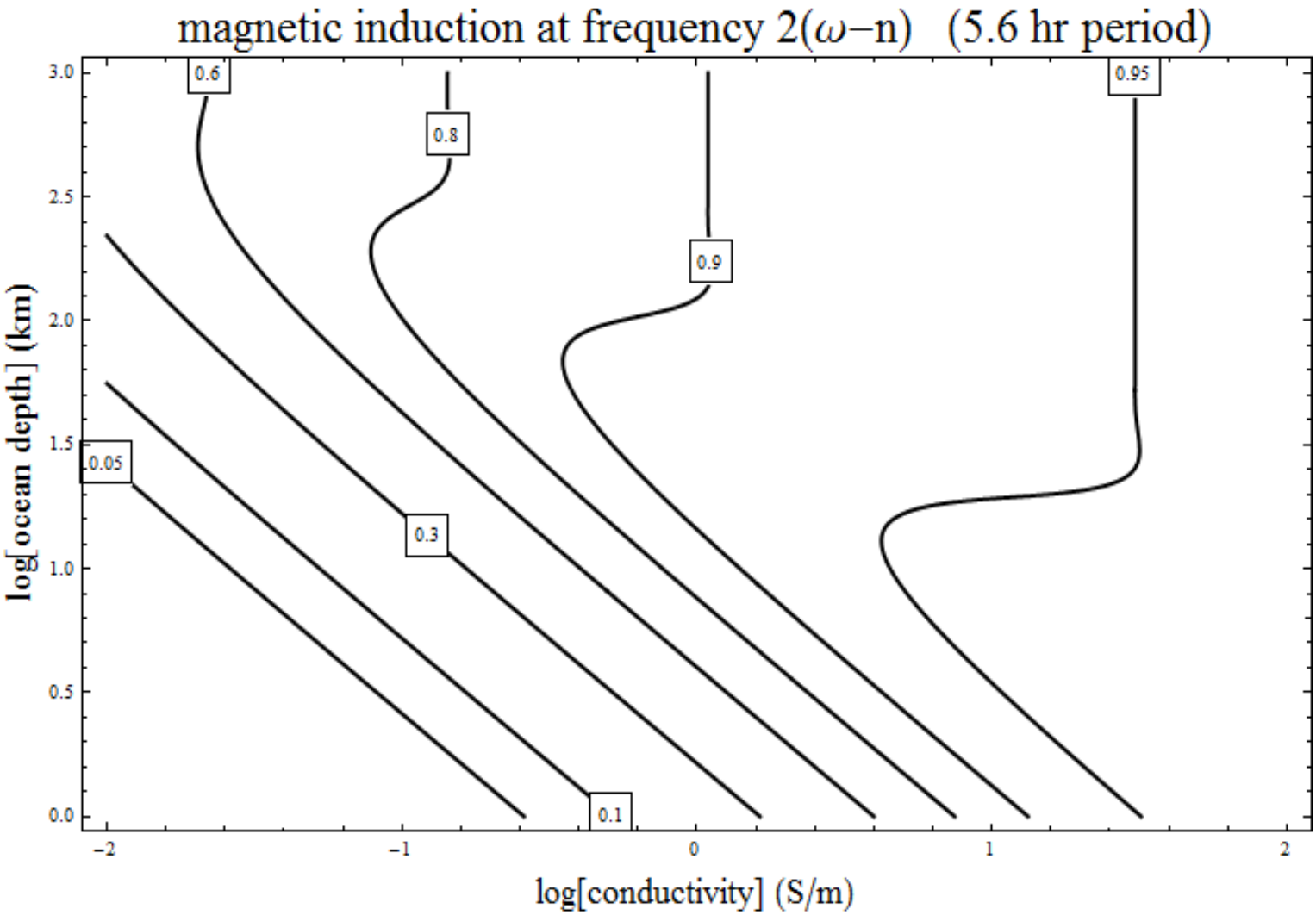
magnetic induction response



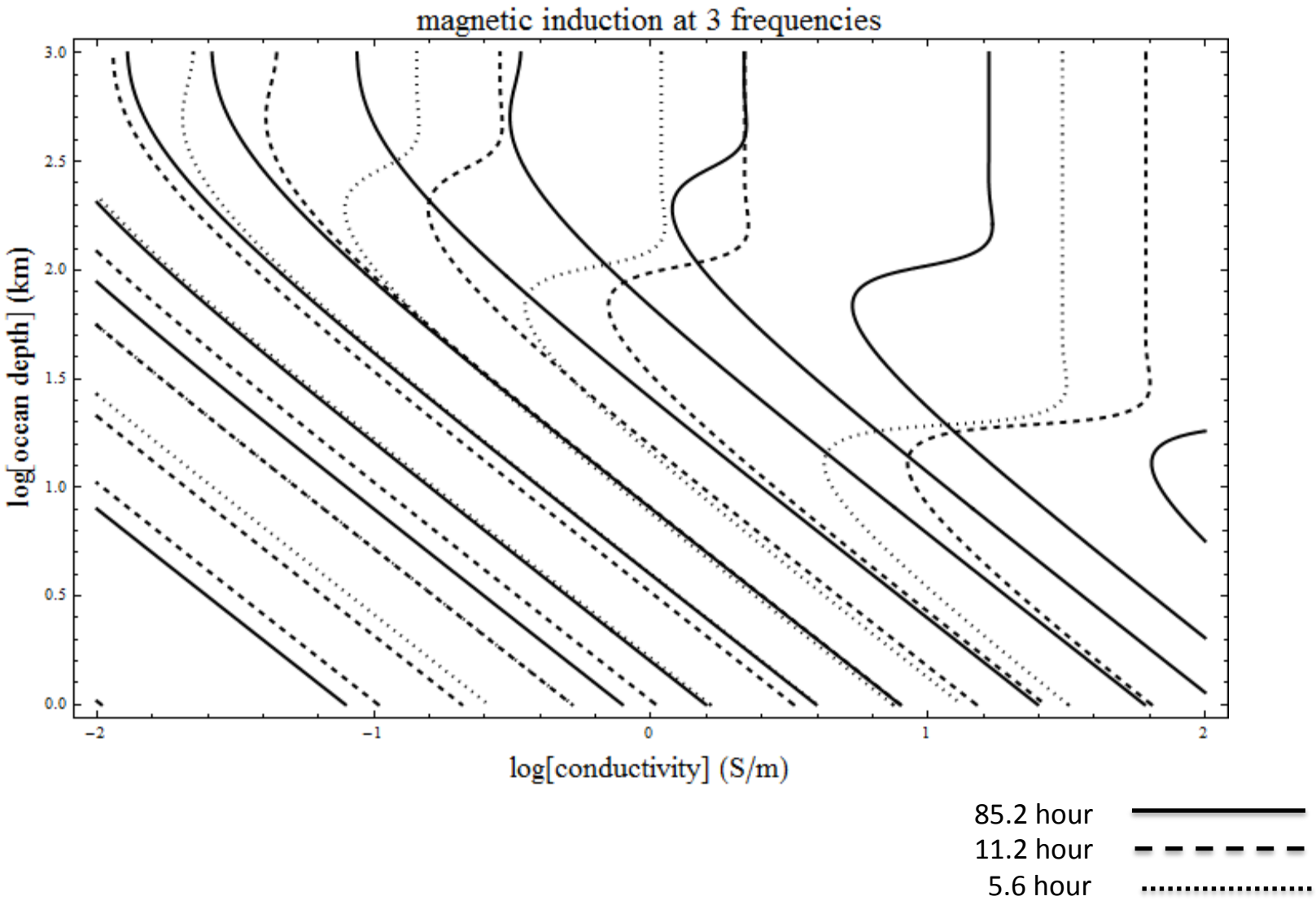
magnetic induction response



magnetic induction response

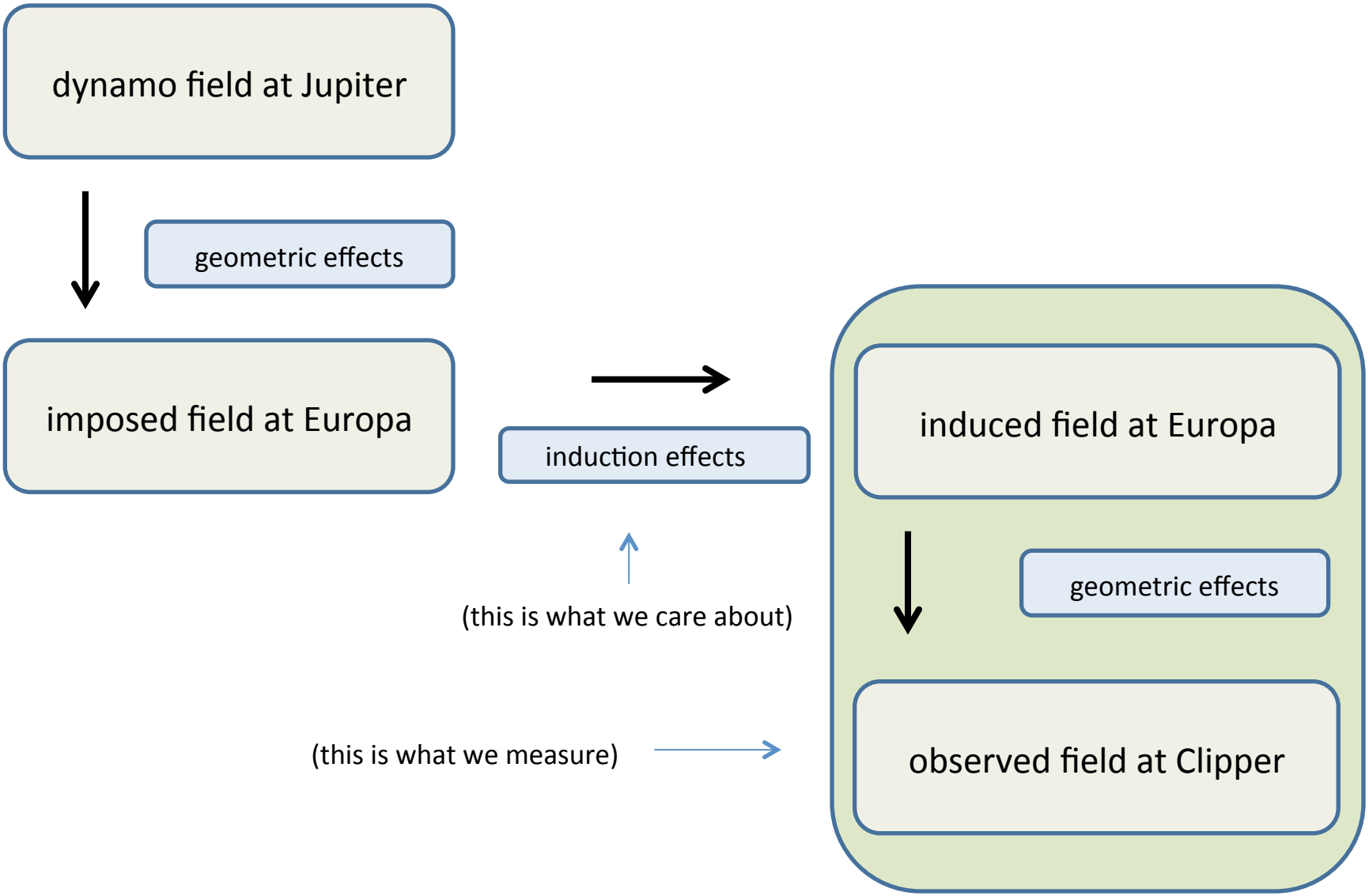


magnetic induction response



magnetic field at Clipper from induced field in Europa

field at Clipper



field at Clipper

The time variations, in the induced Europa field, as seen at Clipper, would be:

$$b_x[t] = 1/2\,(R/r)^3\,(3\,(B\cdot p/p\cdot p)\,p_x - B_x)$$

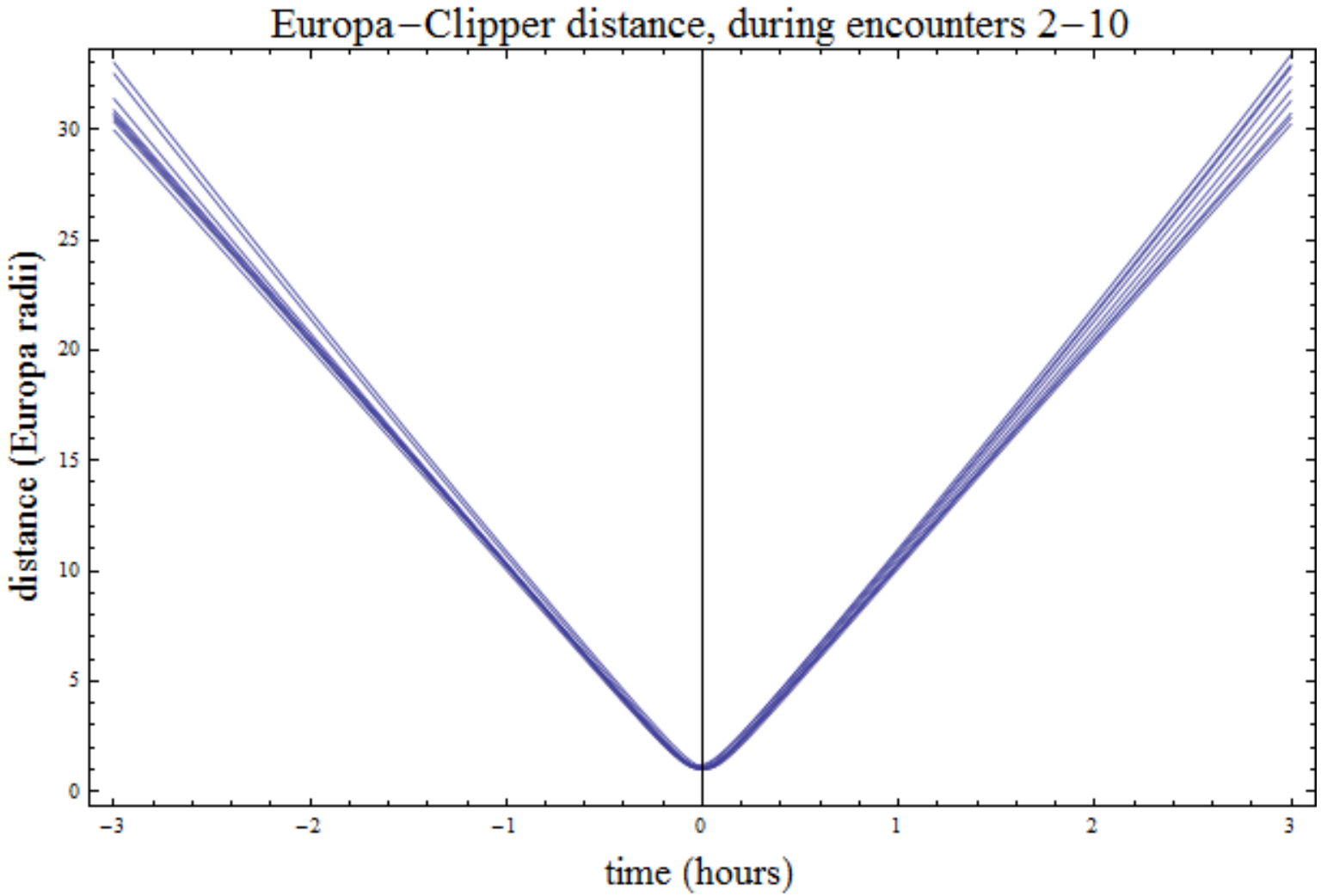
$$b_y[t] = 1/2\,(R/r)^3\,(3\,(B\cdot p/p\cdot p)\,p_y - B_y)$$

$$b_z[t] = 1/2\,(R/r)^3\,(3\,(B\cdot p/p\cdot p)\,p_z - B_z)$$

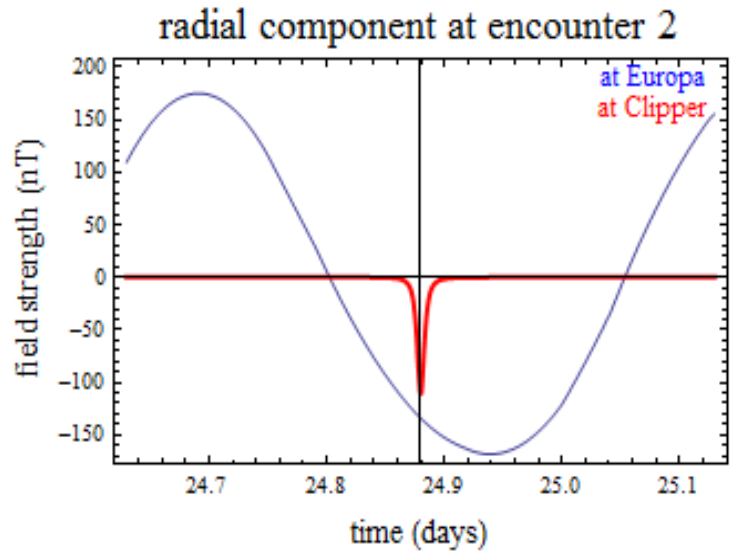
where

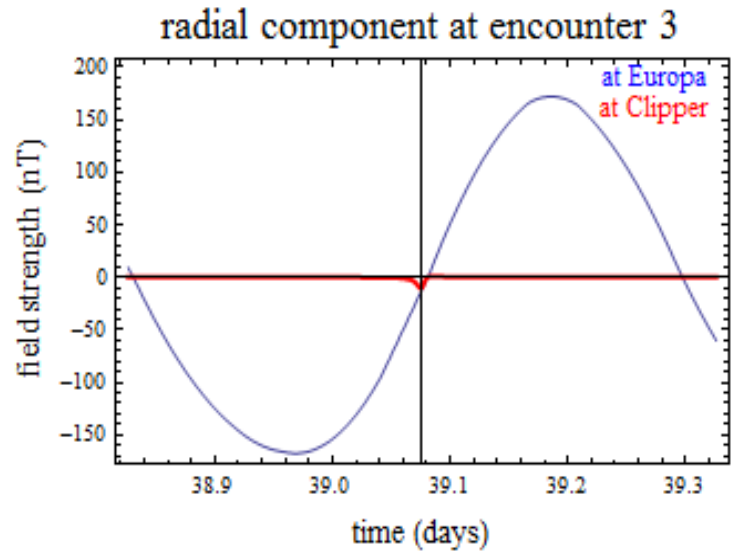
- R is Europa's radius,
- B is induced field at Europa,
- p is position of Clipper, as seen from Europa
- r is Europa-Clipper distance

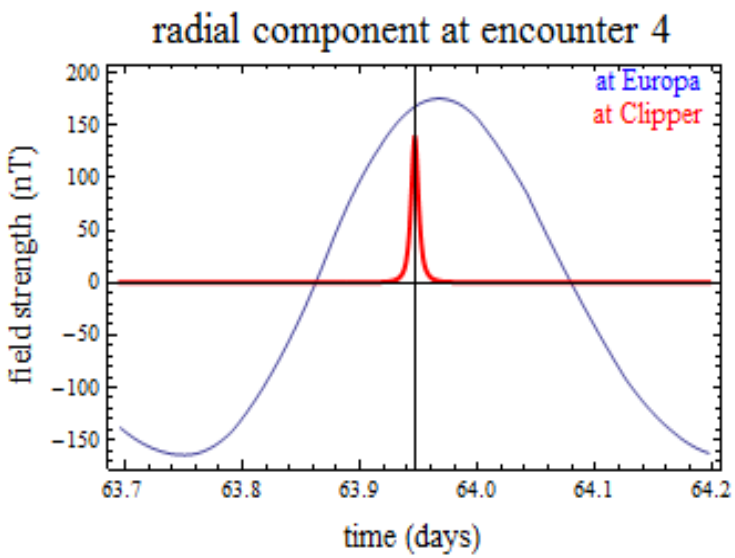
field at Clipper

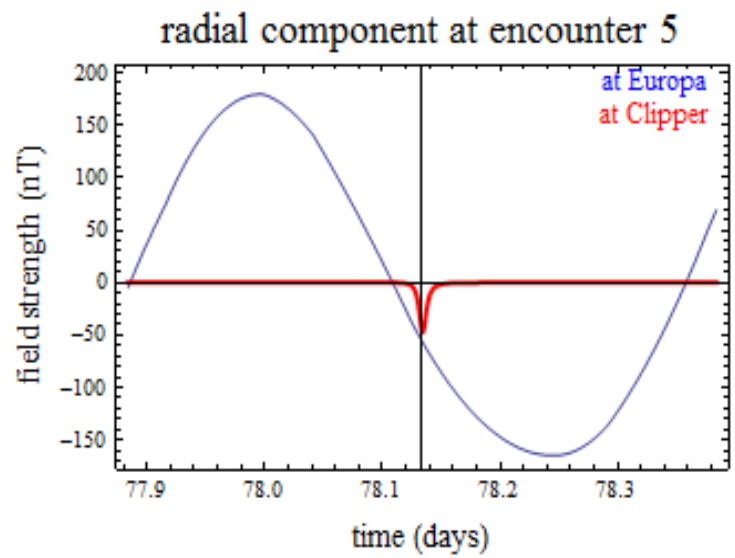


field at Clipper

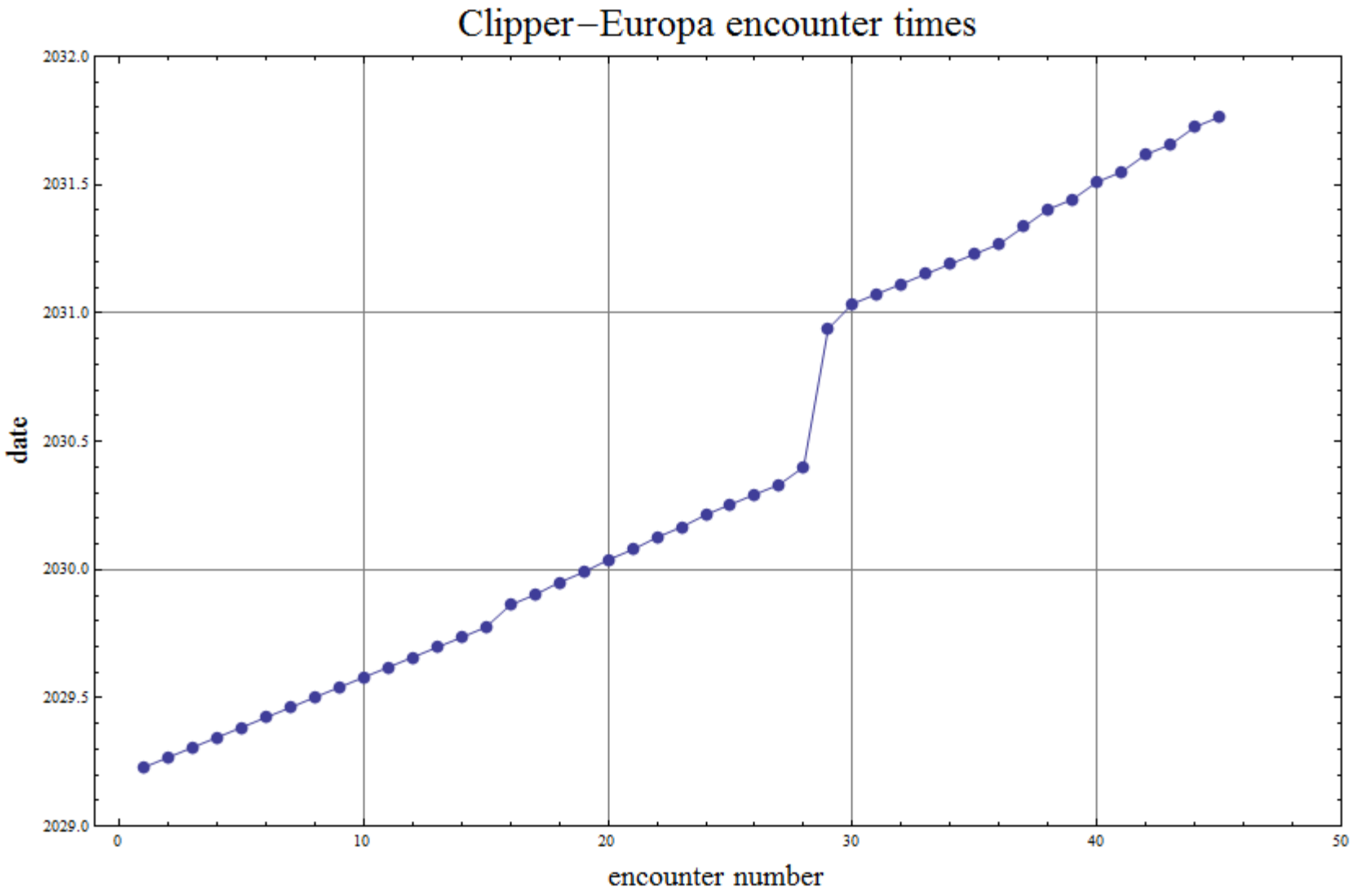








field at Clipper



parameter estimation

Given observations of the magnetic field,
along the Clipper trajectory,

how do we estimate the induction transfer functions Q?

parameter estimation

Based upon Fourier analysis of variations in Cartesian components of imposed magnetic field at Europa, we select 9 base frequencies

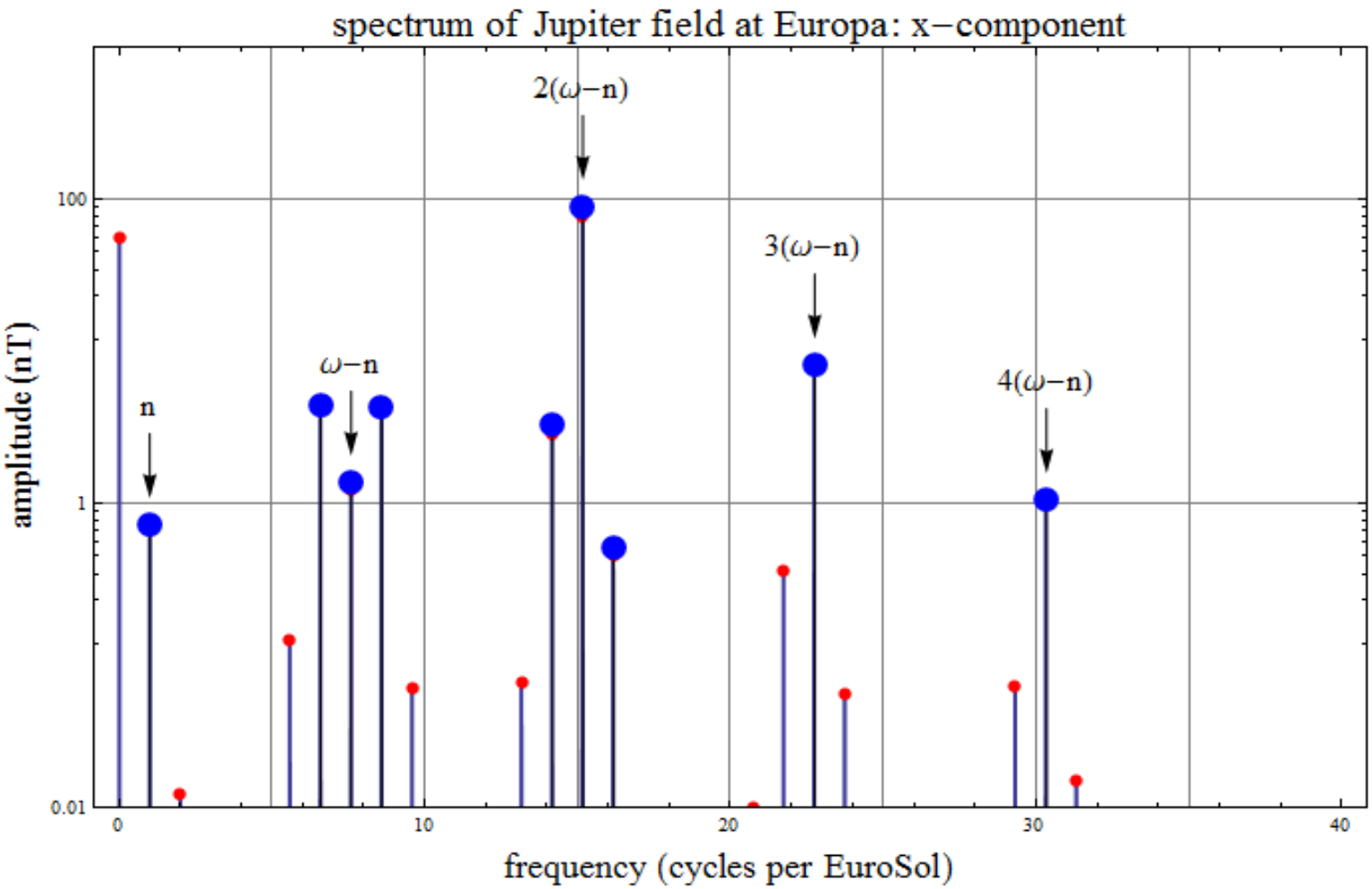
$$\sigma_k = i\,n + j\,\omega$$

where

- n is Europa orbital rate
- ω is Jupiter rotation rate

k	i	j	period (hr)
1	1	0	85.228
2	-2	1	12.938
3	-1	1	11.233
4	0	1	9.925
5	-3	2	6.013
6	-2	2	5.617
7	-1	2	5.269
8	-3	3	3.744
9	-4	4	2.808

parameter estimation



parameter estimation

observation equation:

$z_i = A_{i,j}\ x_j \pm \sigma_i$

where

z_i = i-th observation and σ_i = associated error

x_j = j-th parameter

$A_{i,j}$ = partial derivative of i-th observation with respect to j-th parameter

solution:

$x = (A^t \cdot W \cdot A)^{-1} \cdot (A^t \cdot W \cdot z)$

where the data weight matrix has the form

$W_{i,i} = 1/\sigma_i^2$

the solution covariance matrix is

$C = (A^t \cdot W \cdot A)^{-1}$

and the uncertainty in the estimate of the j-th parameter is

$\varepsilon[x_j] = \sqrt{C_{j,j}}$

parameter estimation

for simulation purposes, our main interest is in the solution **covariance matrix**

$C=(A\uparrow t \cdot W \cdot A)\uparrow -1$

which yields

- uncertainty in estimate of j-th parameter:

$\varepsilon[x\downarrow j\,]=\sqrt{C\downarrow j,j}$

- correlation between parameters "*i*" and "*j*" :

$$\rho_{i,j} = \frac{C_{i,j}}{\varepsilon[x_i]\,\varepsilon[x_j]}$$

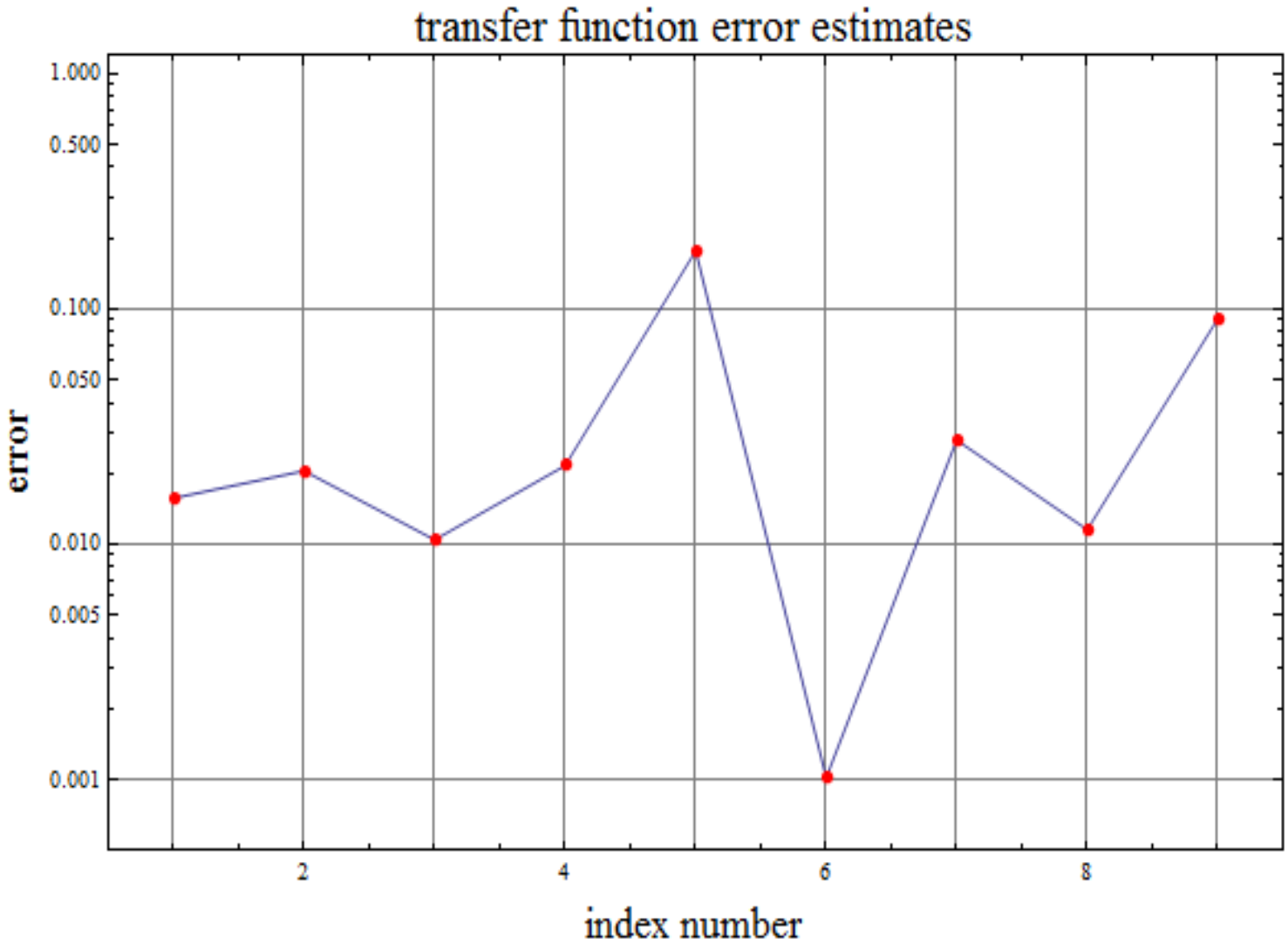
parameter estimation

At each of 45 Europa-Clipper encounters

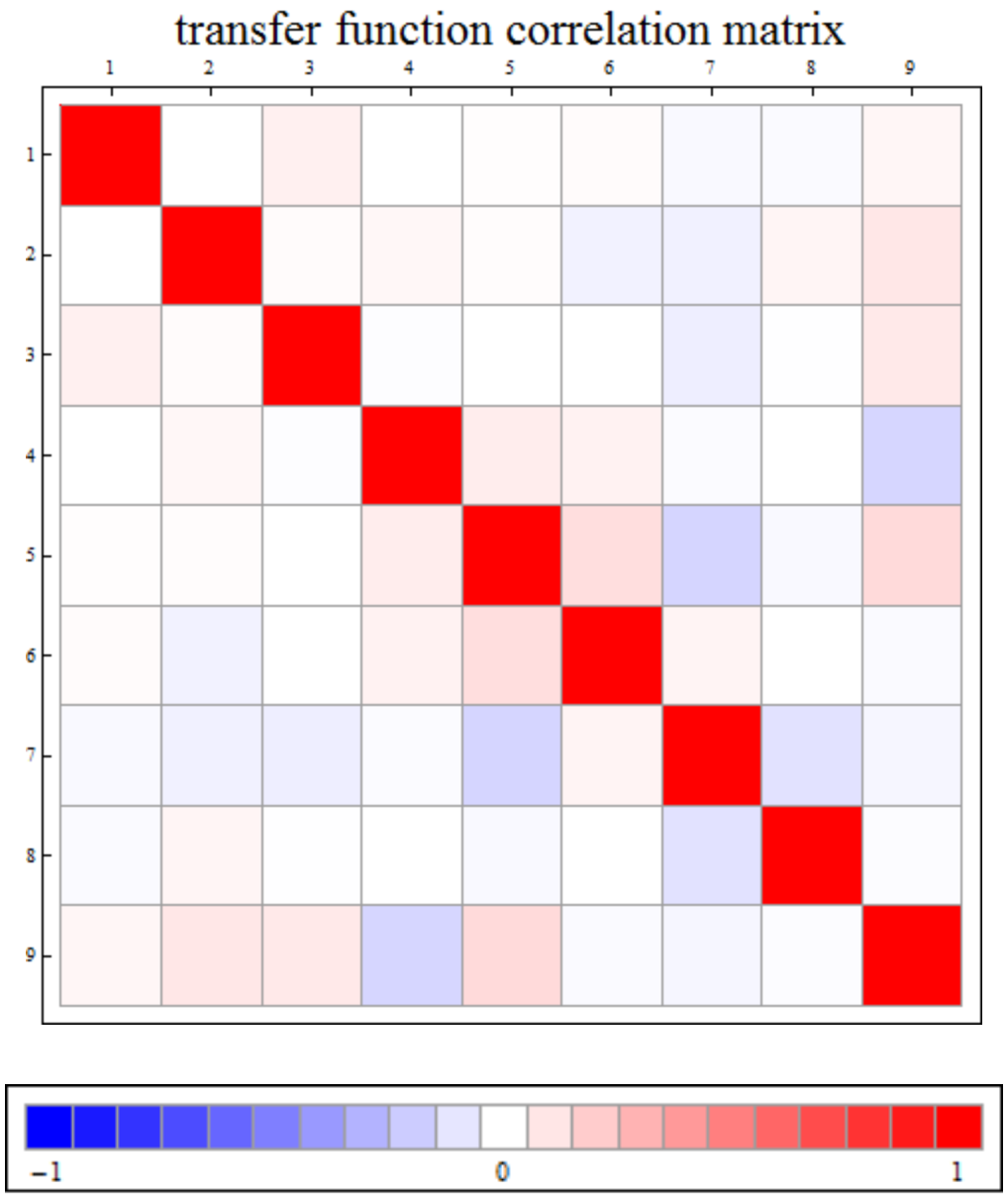
- compute partial derivatives
  - of Europa induced field
  - at Clipper location
  - with respect to each of 9 transfer functions

- technical details
  - 2 hour time span, centered on time of closest approach
  - one vector sample (x,y,z components) each minute
  - assume 1 nT error on each measurement
  - assume errors are uncorrelated (MORE ABOUT THIS BELOW)

parameter estimation



parameter estimation



summary to this point

These simulations suggest that the
Europa magnetic induction experiment
could be adequately conducted from the considered
series of 45 flyby encounters.

For the 9 attempted transfer functions,
accuracy is good

- 1 at 0.1 %
- 7 between 1% and 10 %
- 1 worse than 10%

Pair-wise correlations are low

- all are below 0.2

Multiple well-determined transfer functions help
better resolve the internal conductivity structure
of Europa.

long period correlated error

- sources of magnetic field variation:
  - deterministic variation, due to
    - orbital motion of Europa, and
    - rotation of Jupiter
  - broad-band stochastic variation, due to
    - instabilities in Jovian magnetosphere, and
    - plasma waves
- previous analysis, for Europa-Clipper
  - only included deterministic effects

long period correlated error

assumed error variance spectrum:

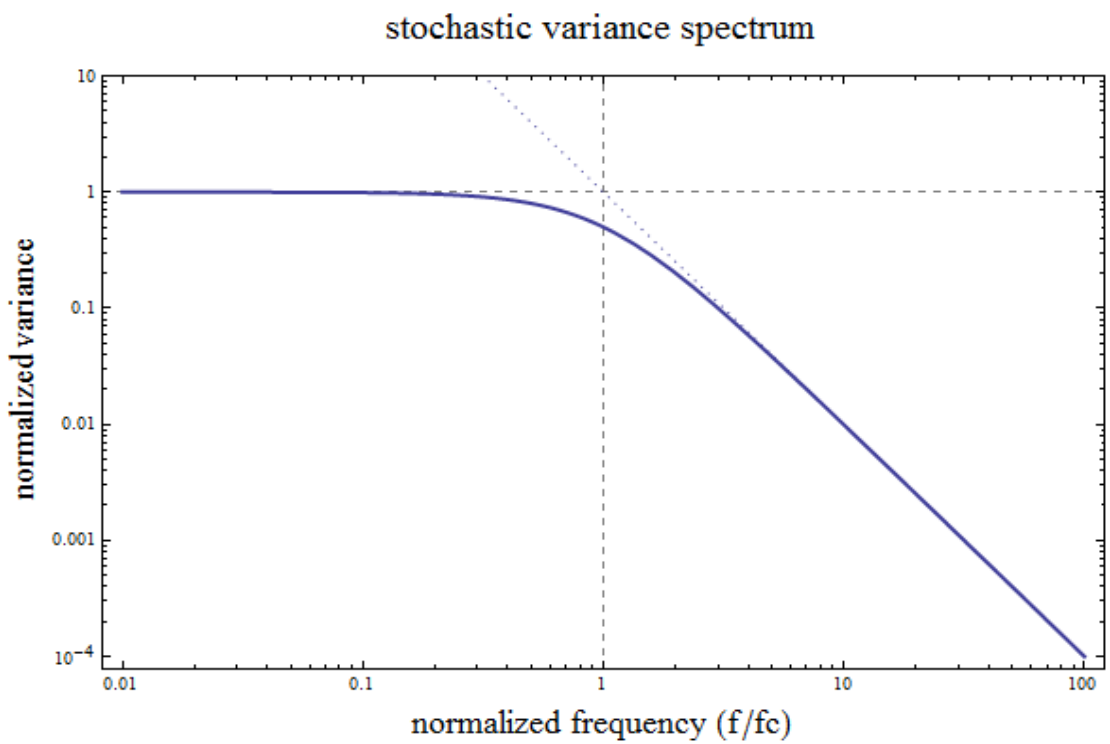
$$V[f] = b^2 / 1 + (f / f_c)^2$$

where

b is a scale factor (nT)

f is frequency of variation

f_c is a "corner frequency"



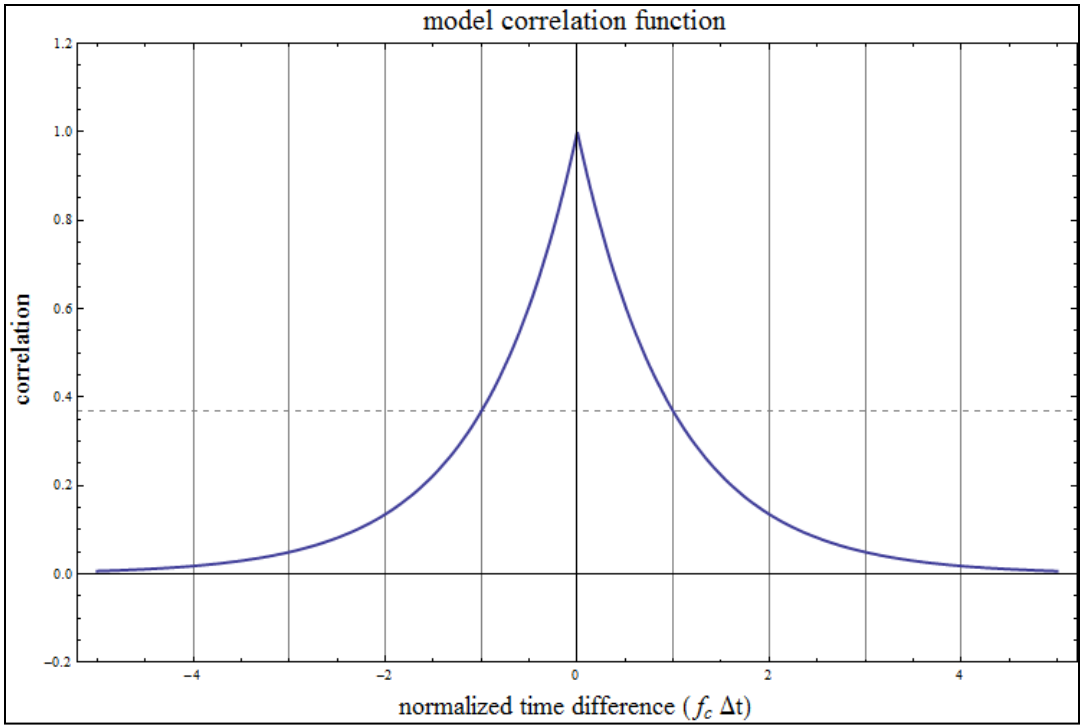
long period correlated error

associated covariance function:

$$\mathrm{Cov}[\Delta t] \equiv E[B[t] * B[t+\Delta t] = \sqrt{\pi/2} \quad b\uparrow 2 \;\; f\downarrow c \;\; exp[-f\downarrow c\,|\Delta t|]$$

and correlation function:

$$\mathrm{Cor}[\Delta t] \equiv \; Cov[\Delta t]/Cov[0] \;\; = exp[-f\downarrow c\,|\Delta t|]$$



long period correlated error

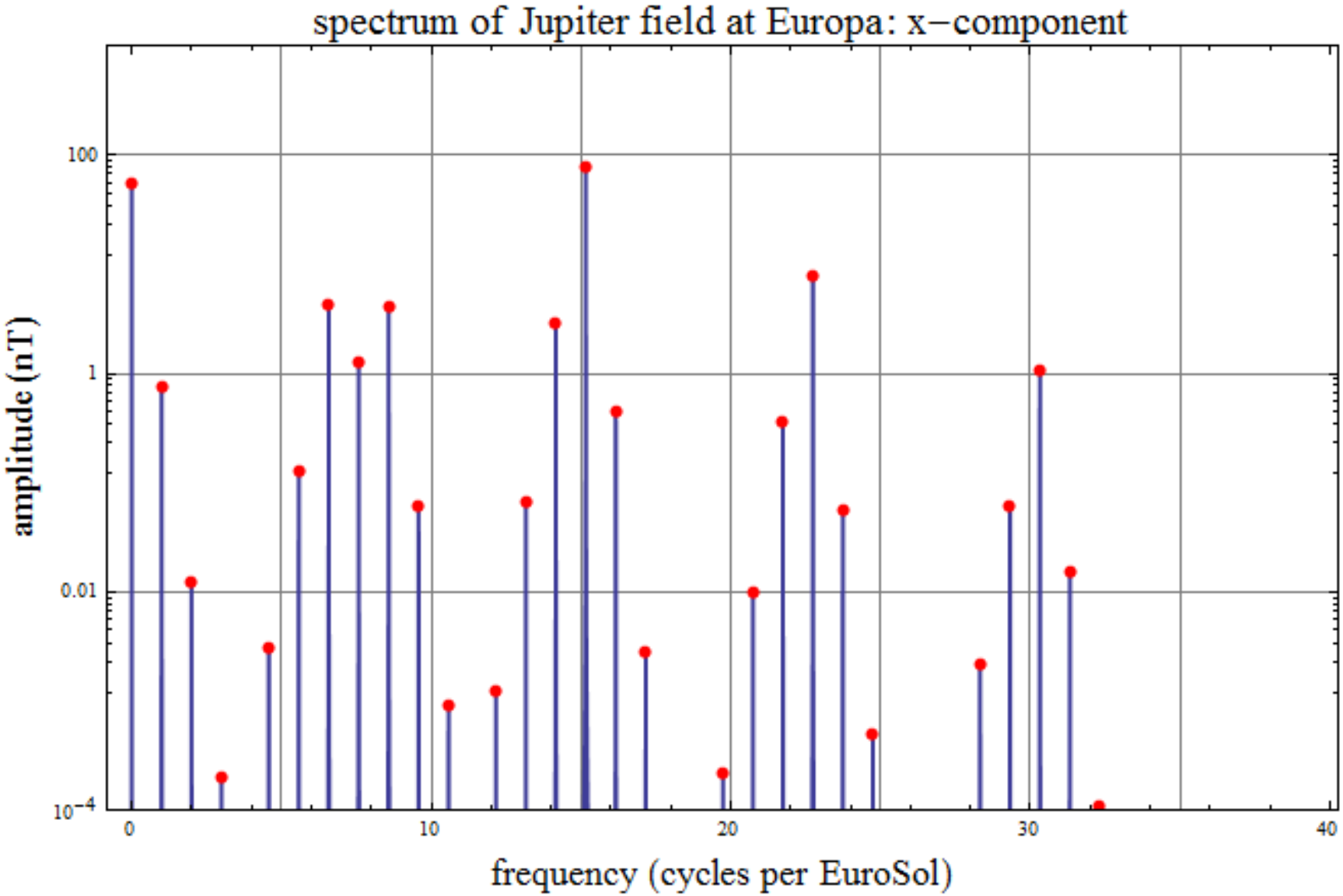
based on Galileo mission data analysis:

current best estimates are

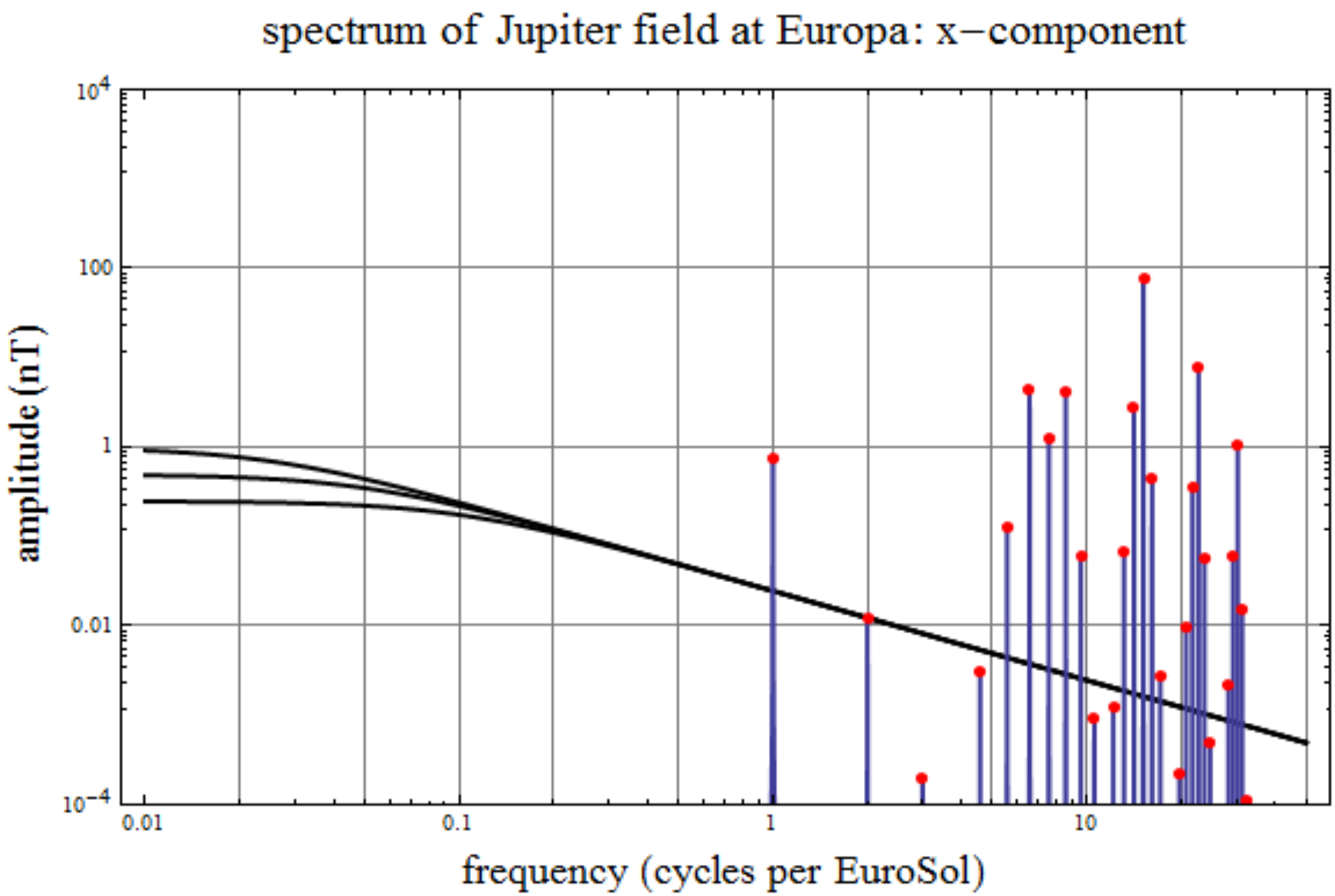
amplitude: b = 20 nT

de-correlation time: $2\pi/f\downarrow c \sim 30\ days$

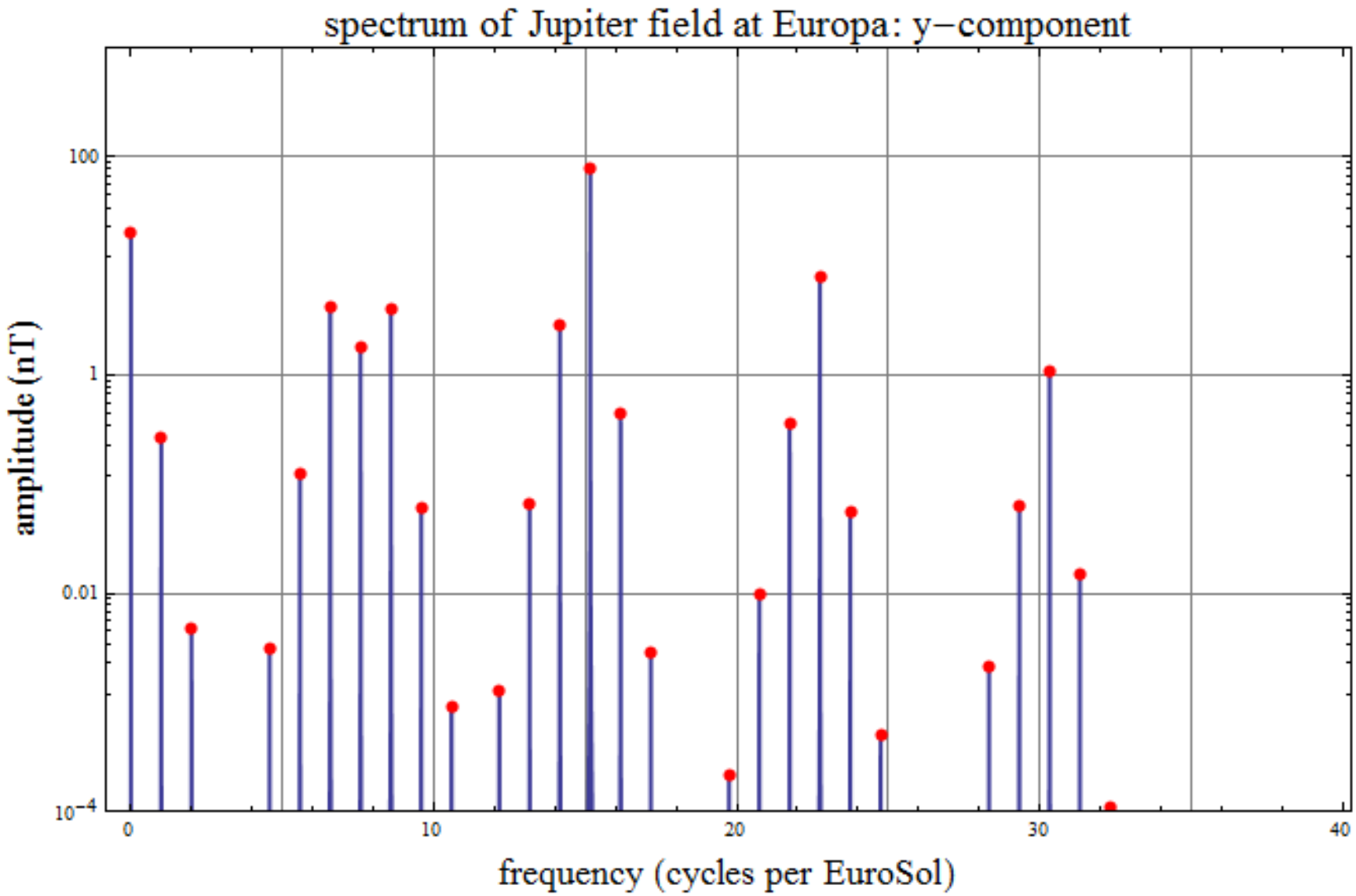
without background noise



with background noise



without background noise



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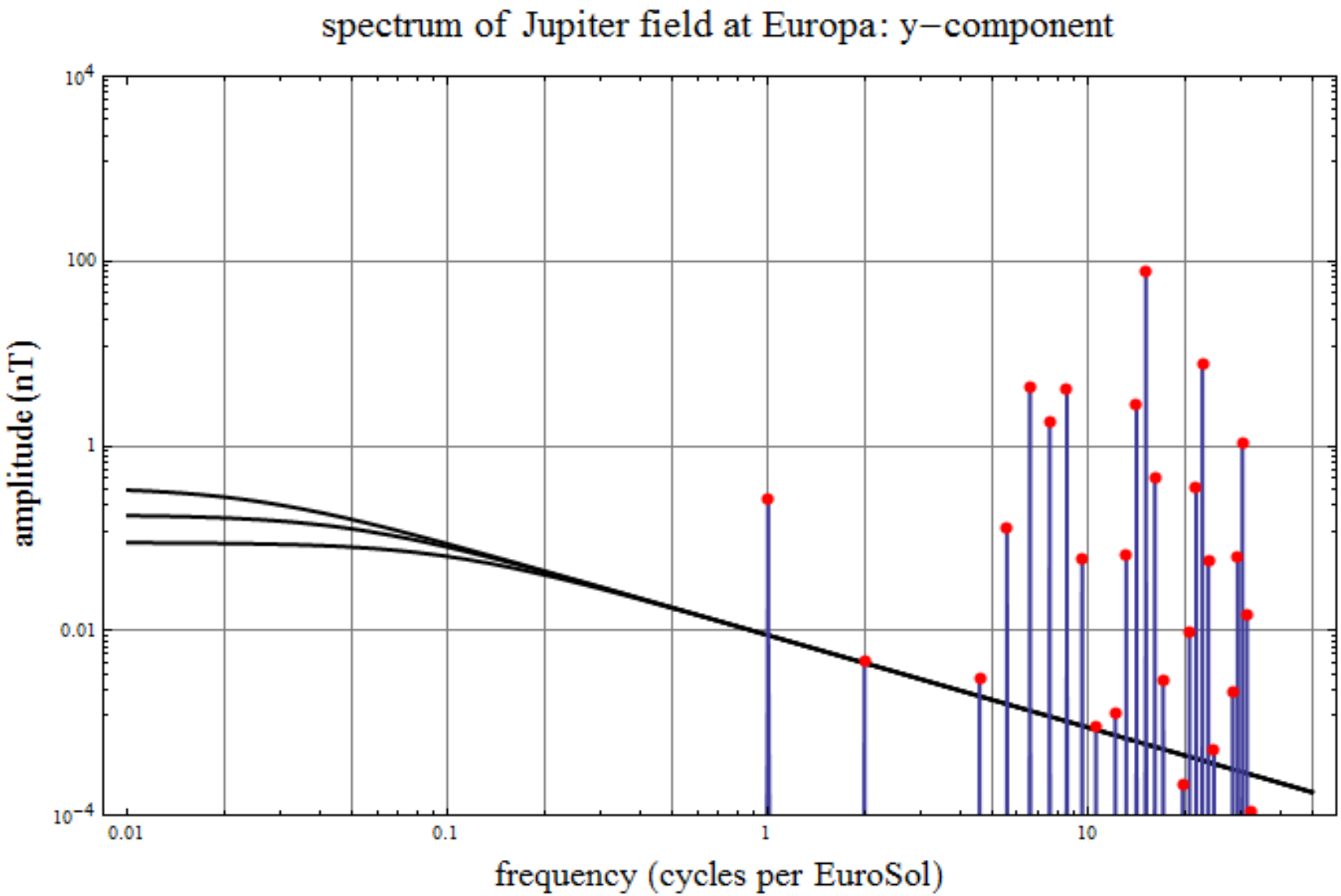
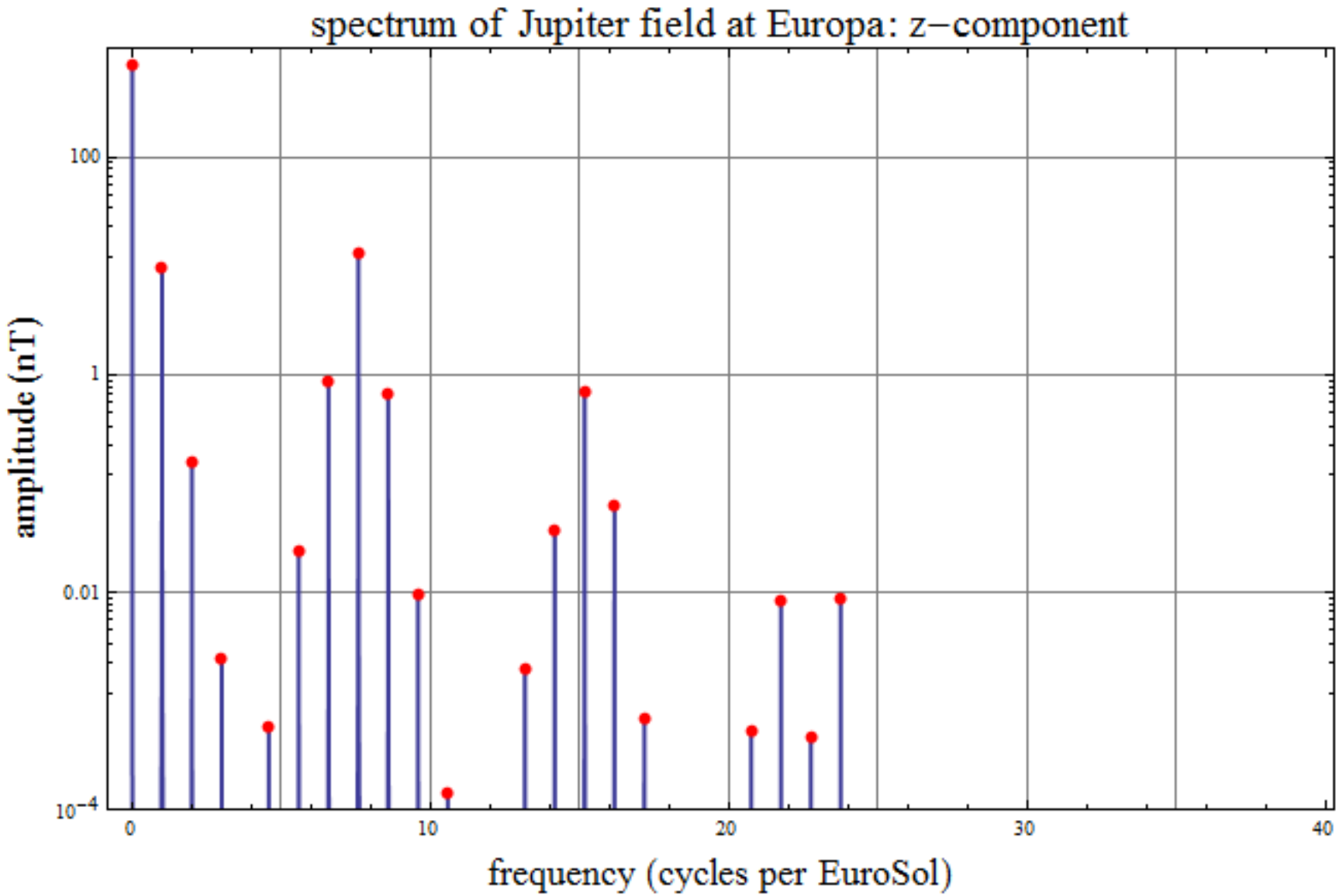
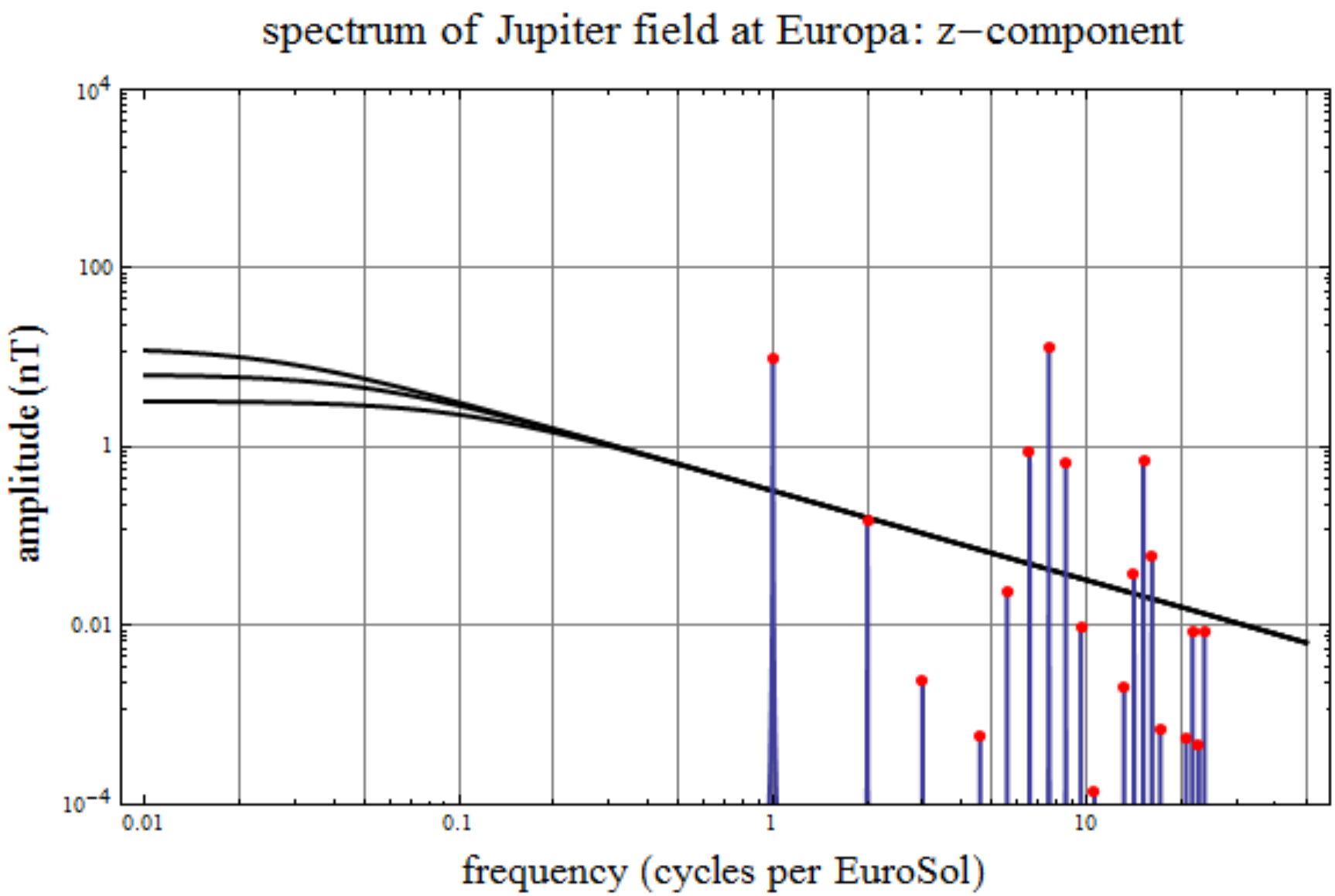


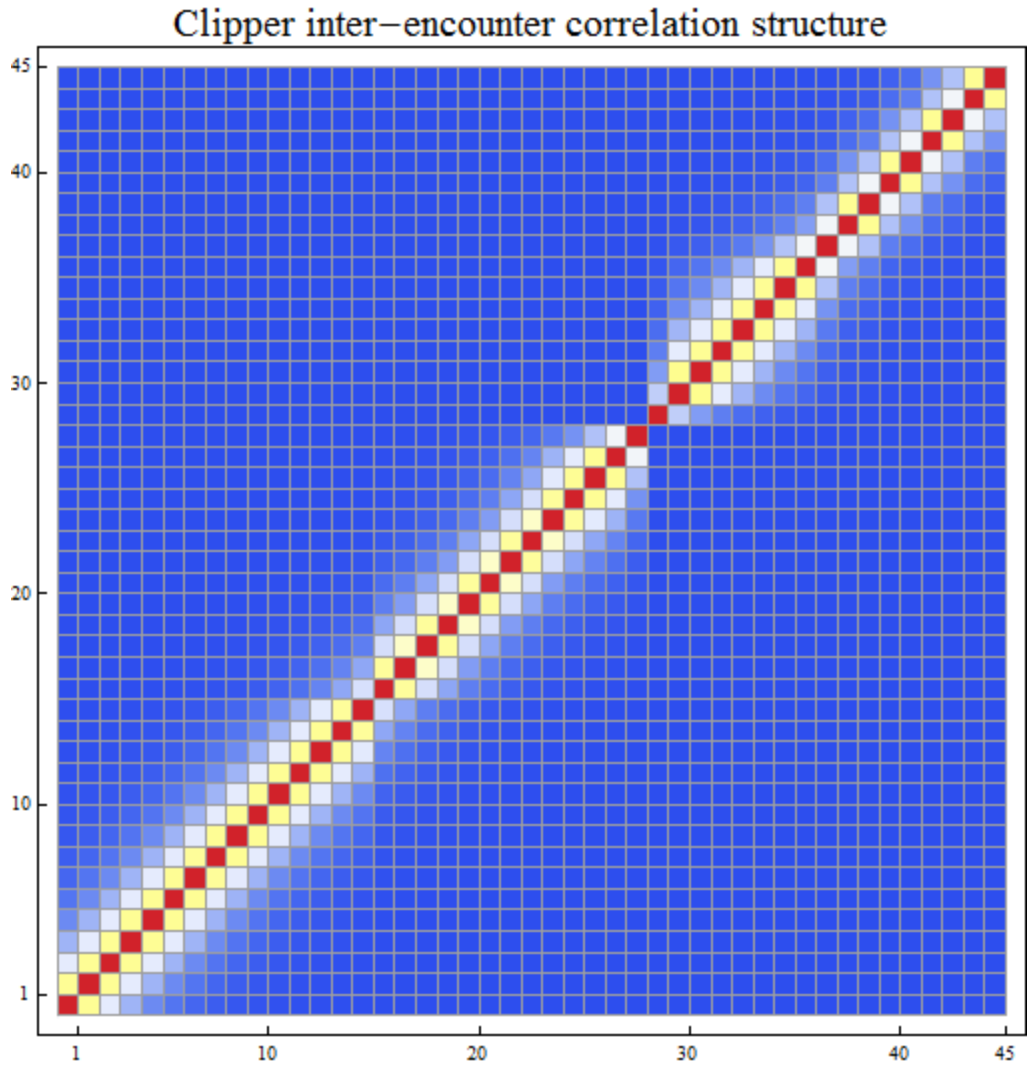
without background noise



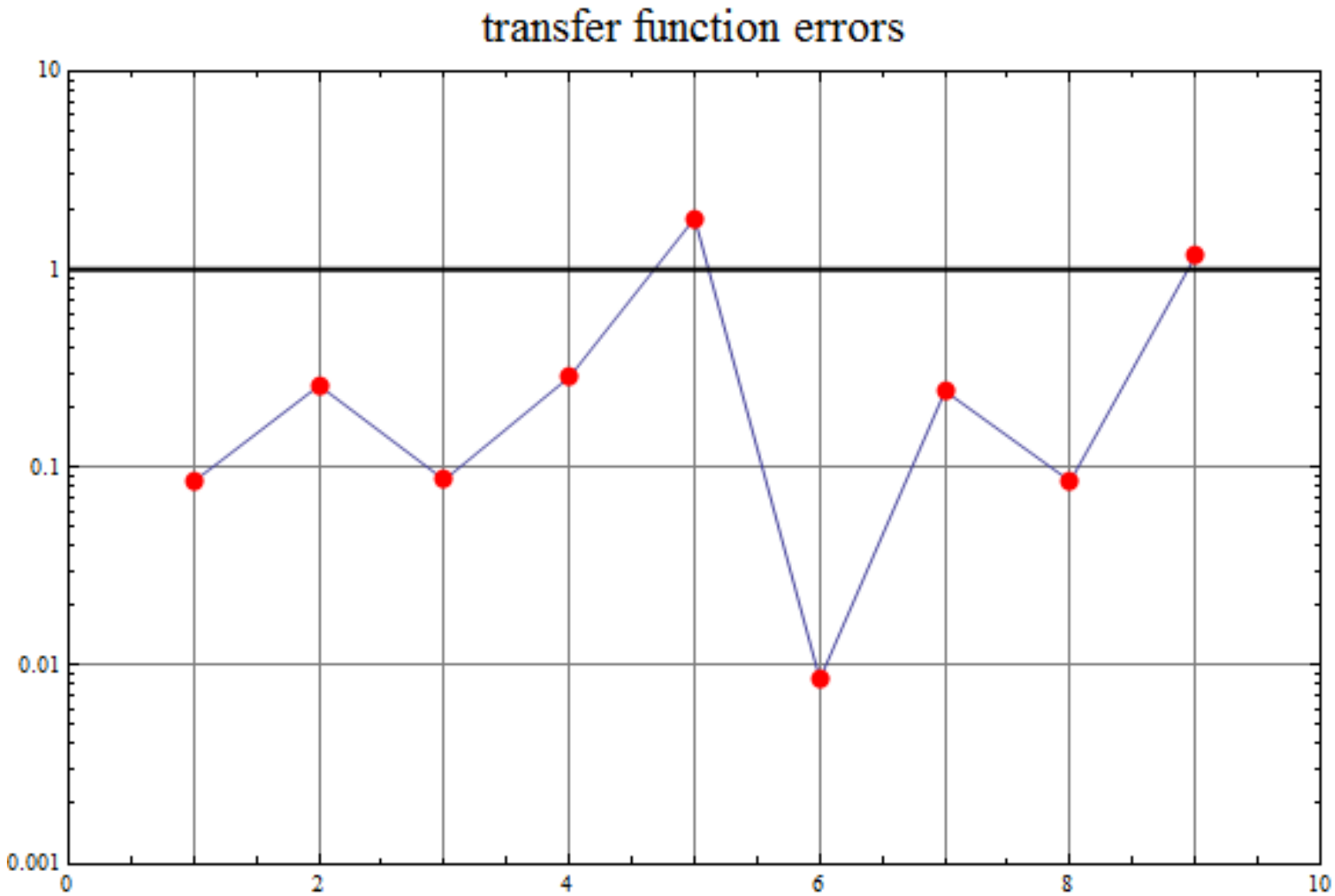
with background noise



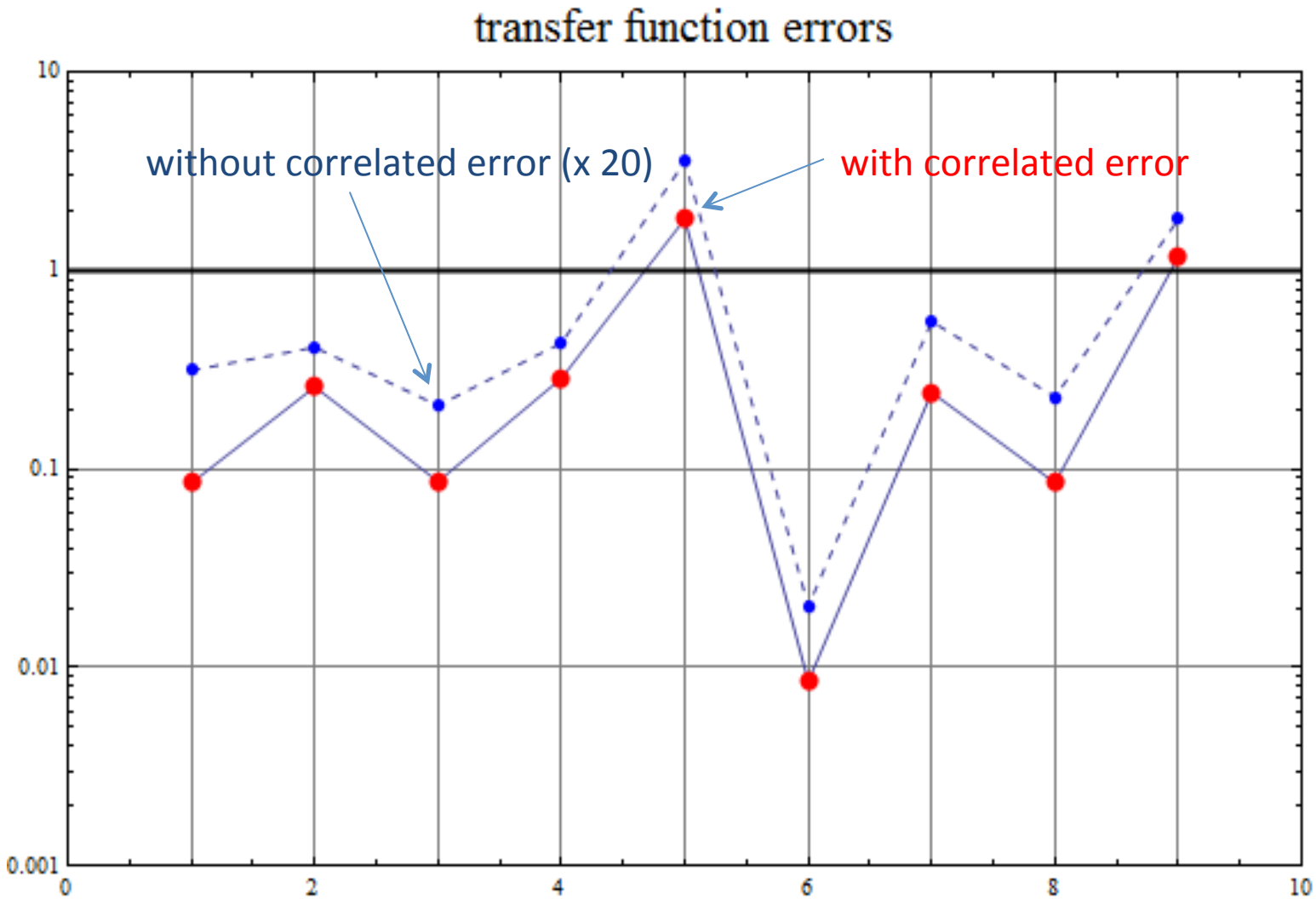
long period correlated error



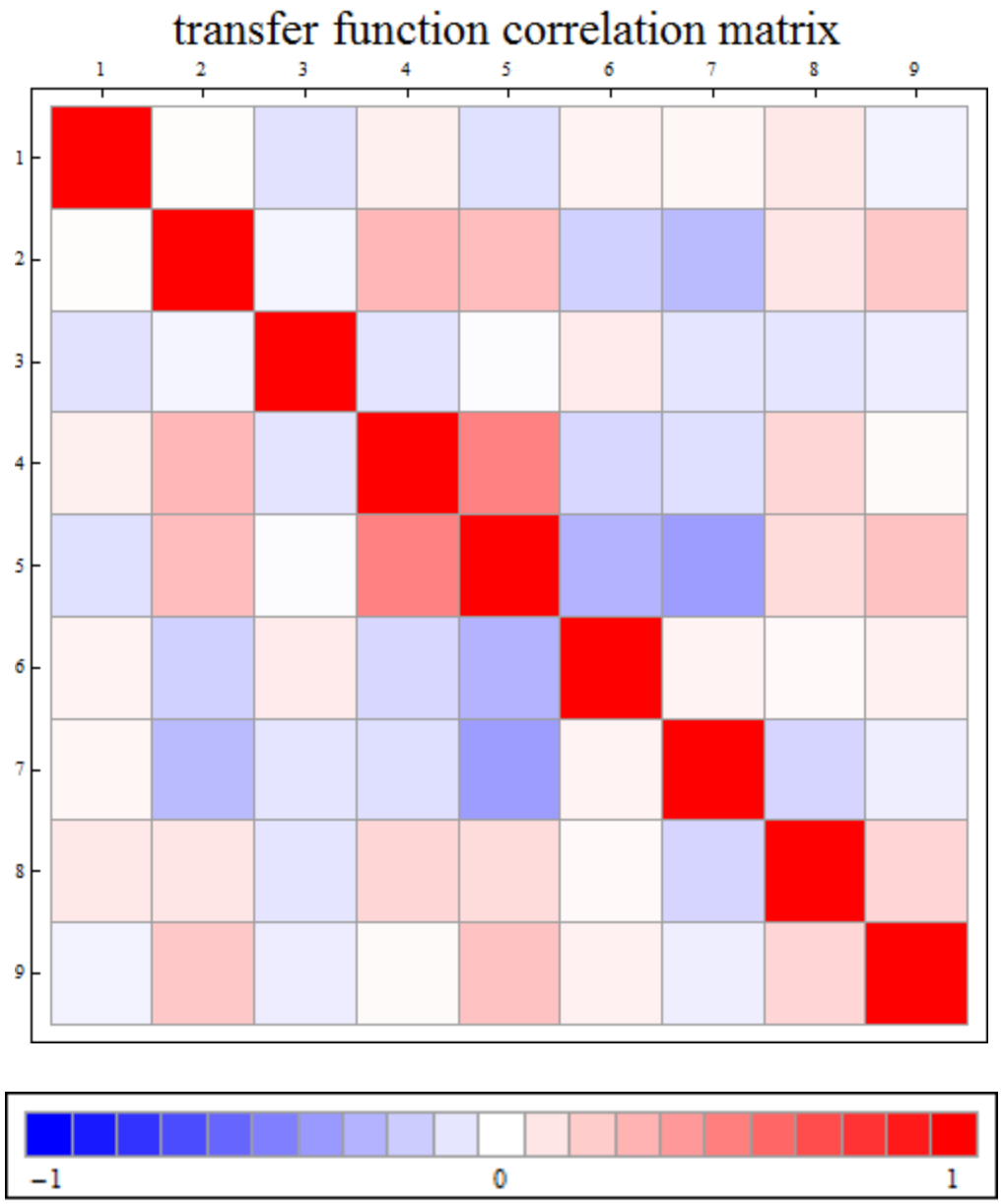
long period correlated error



long period correlated error



long period correlated error



long period correlated error

Summary of correlated error effects

assuming the parameter values used in the present simulation, which are

- scale factor of 20 nT
- decorrelation time of 30 days

the recovered induction response, from 45 Clipper encounters, is largely the same as would be obtained

- without correlated noise, but
- assuming a single measurement error of 20 nT

In particular, 7 of the 9 attempted transfer functions are recovered with useful accuracy

long period correlated error

Conclusions

- Simulation of the Europa Clipper magnetic induction experiment suggest that it should be successful at resolving responses at many different forcing frequencies.

- This is due to the fact that the encounter sequence spans an interval of time which is long, compared to the background noise de-correlation time.

- By implication, a shorter duration orbiter mission would not do as well.