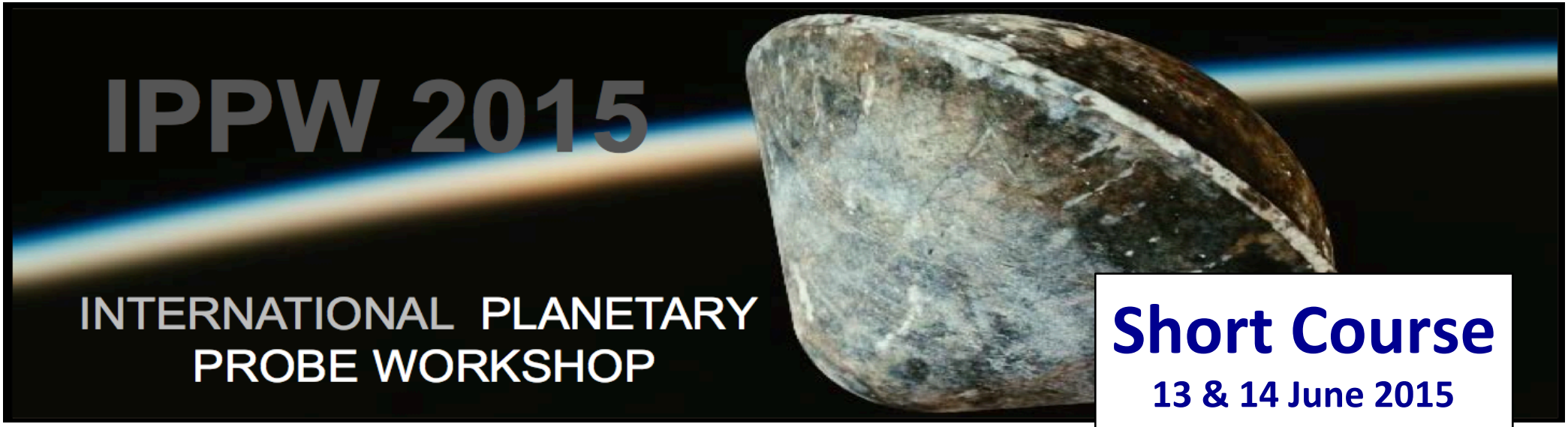

Short Course
13 & 14 June 2015
Cologne, Germany

Gravity Science

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Tom Andert
Uni. BW, Munich

Jet Propulsion Laboratory
California Institute of Technology

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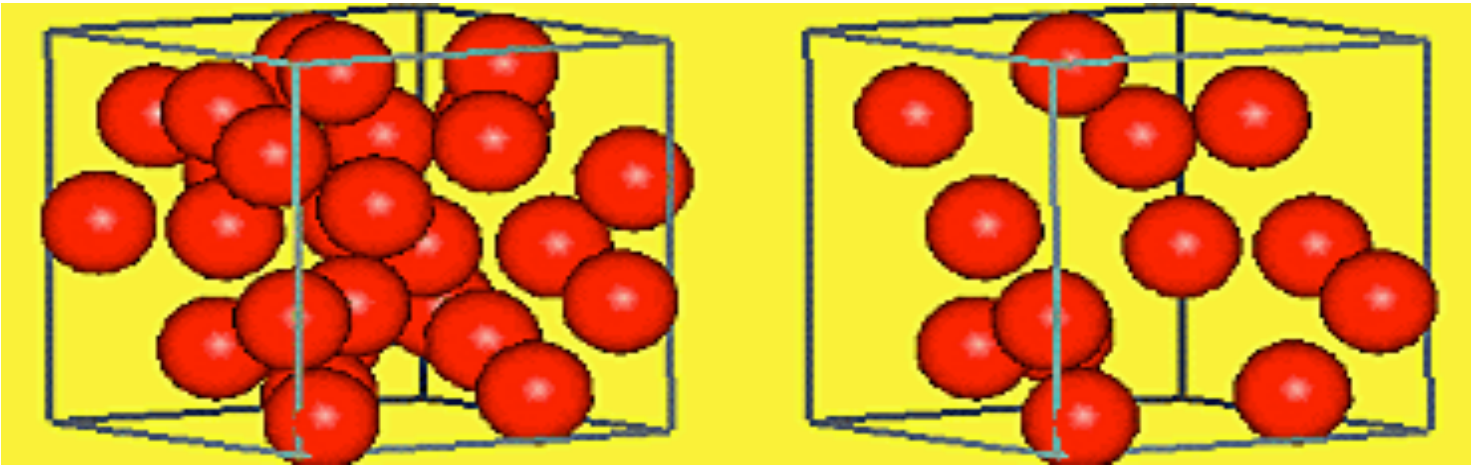
Government sponsorship acknowledged.

OVERVIEW

Sami Asmar

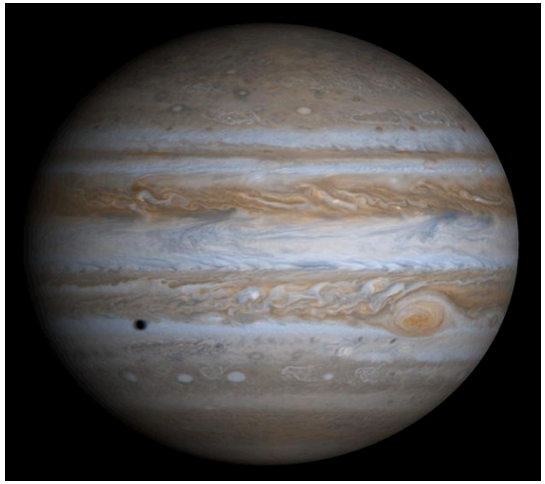
Density & Gravity

- Box on the left has more stuff than one on the right, of the same size
- It has higher density (density = mass / volume)
- Gravity is an attractive force that is proportional to mass inside objects
- For these boxes, the gravity is different because the density is different
- Gravity also depends on distance away from the center of mass



Planets & Gravity

- Jupiter has much more mass than Earth and much more gravity
  - But a lot less density
- At its equator, Jupiter's surface gravity is only 2.5 times Earth's surface gravity because Jupiter is so big
- Dependence on total mass inside the body and the distribution of mass



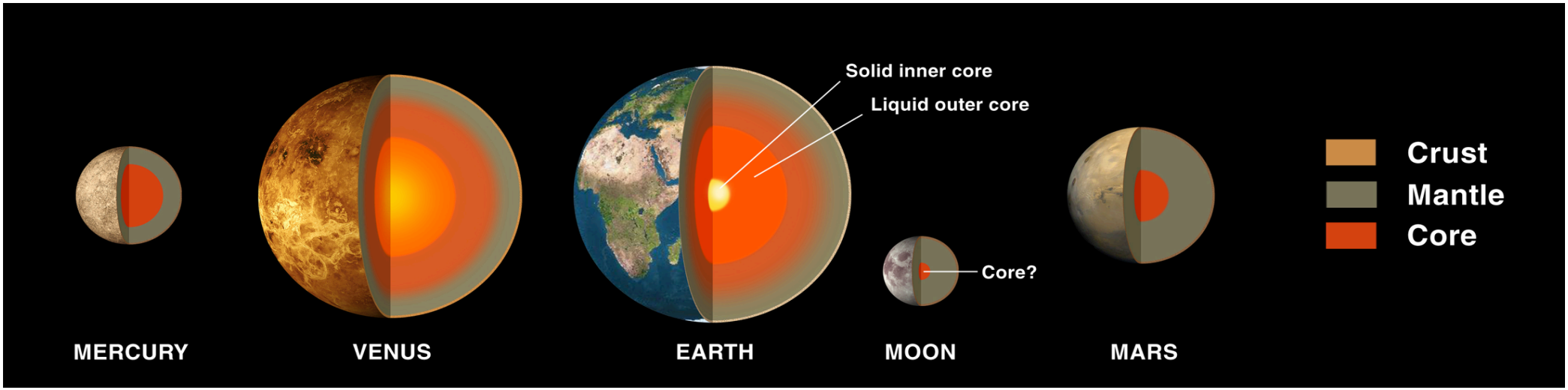
Jupiter:
Diameter: 139822 km (~ 11 x Earth)
Mass: 1.8986×10^{27} kg (~ 317 x Earth)
Average Density: 1326 kg/m^3
Surface Gravity: ~ 2.4 g (~ 23 m/s^2)

Earth:
Diameter: 12742 km
Mass: 5.97219×10^{24} kg
Average Density: 5515 kg/m^3 (~ 4.2 x Jupiter)
Surface Gravity: 1 g (9.8 m/s^2)

Saturn’s density is less than water, so it floats

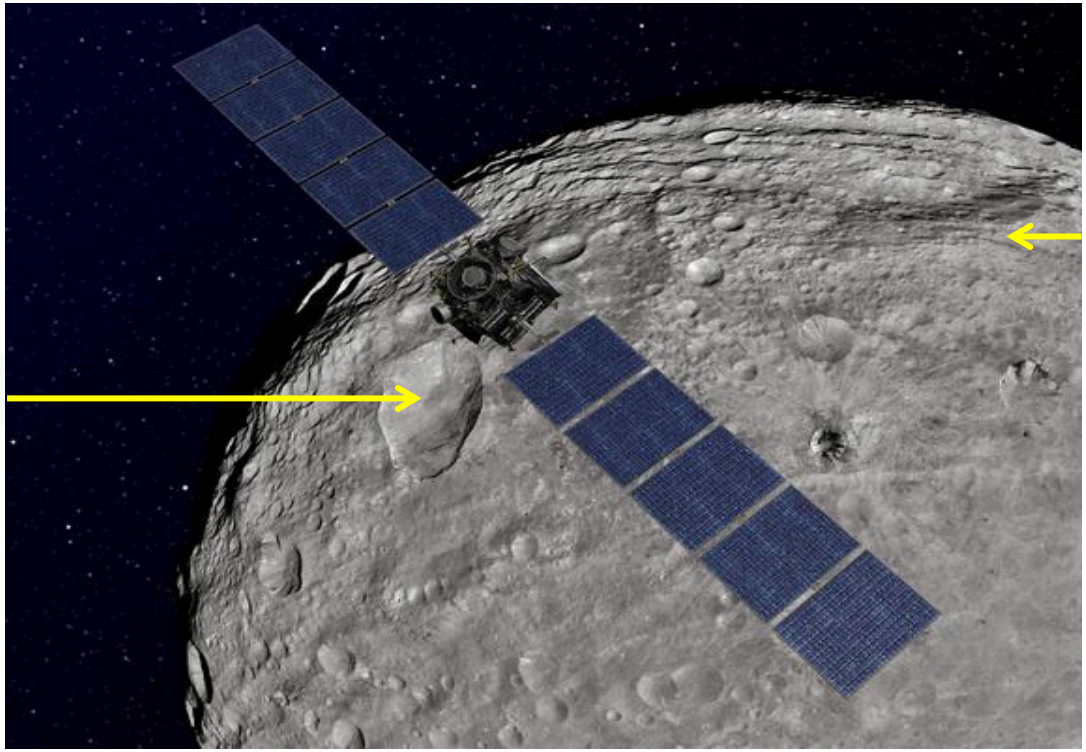
Interior Models

- **Probe inside the planets by determining their gravity fields**



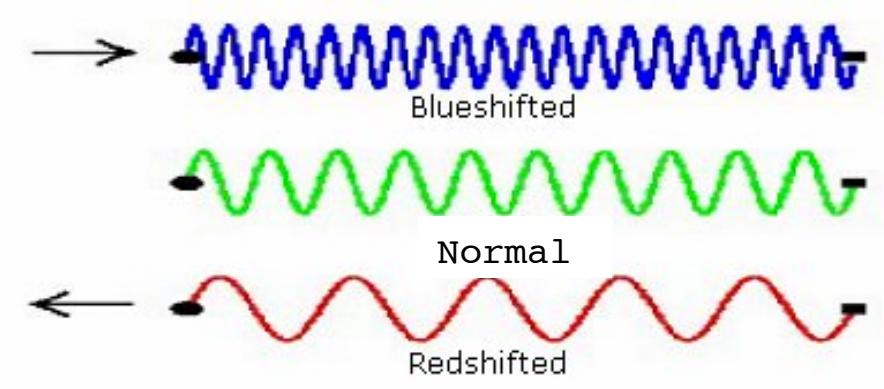
Use Spacecraft to Observe Gravity

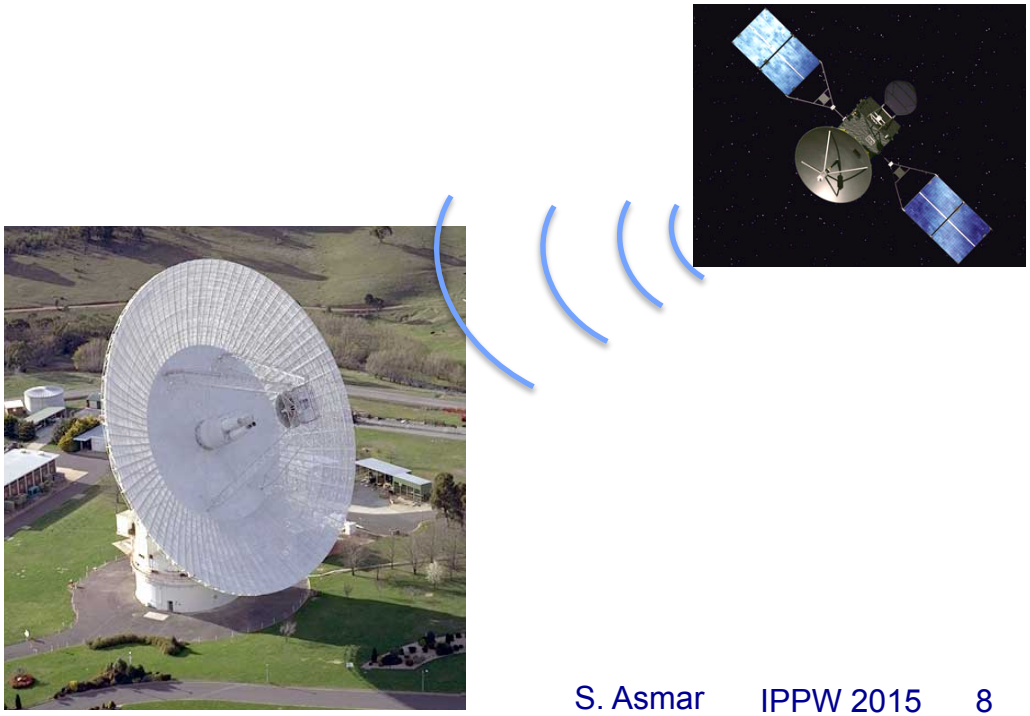
- **Watch the changes in path as a spacecraft orbits around a planet**
- **Speed changes depending upon where it is around the object**
- **Manifested as a Doppler shift in the radio signal**



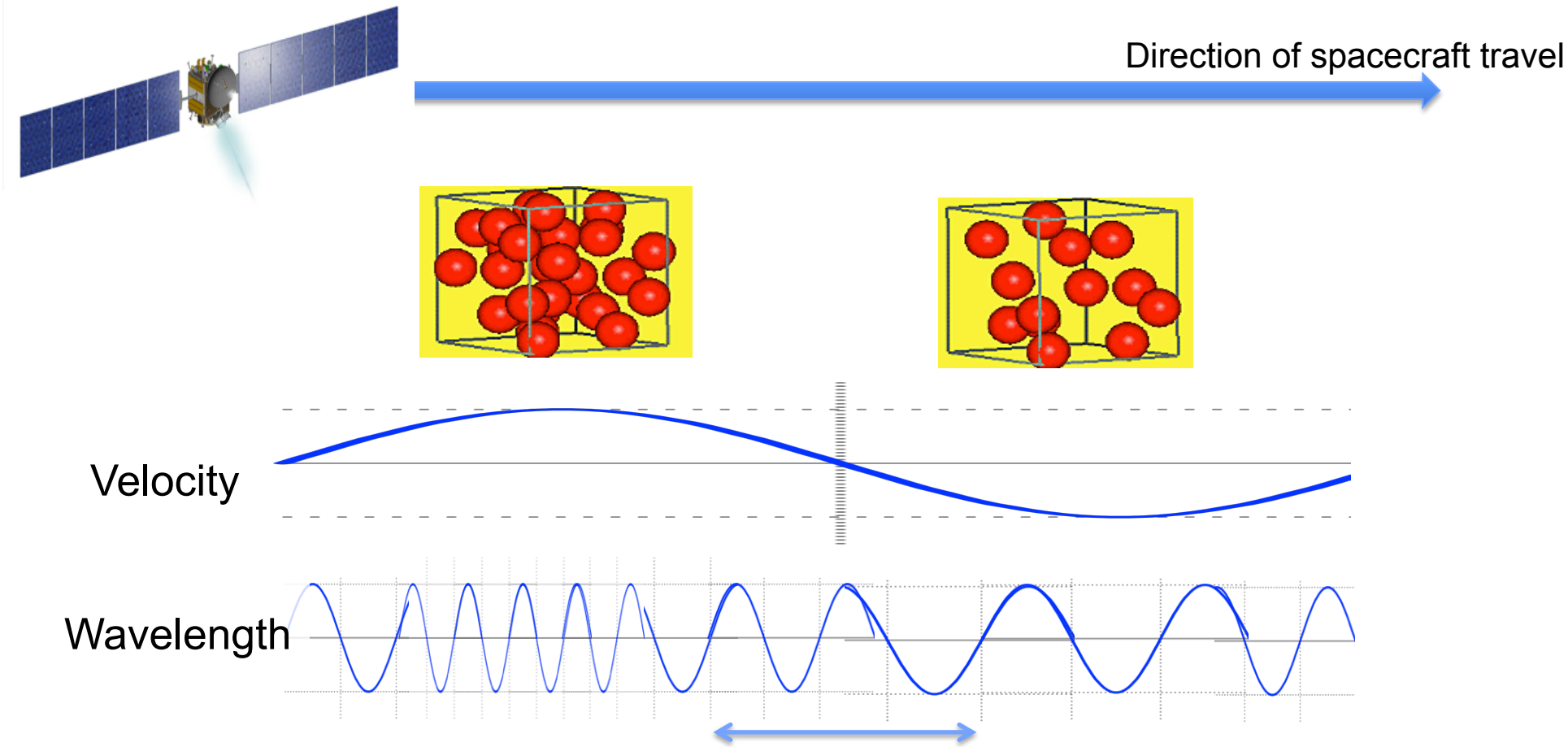
How to Measure Spacecraft Velocity

- Receive spacecraft radio signal on Earth
- As spacecraft speeds up or slows down, this changes the wavelength (frequency) of the received signal
  - Blue shift: wavelength gets shorter. . . when it travels towards Earth
  - Red Shift: wavelength gets longer. . . when it travels away from Earth



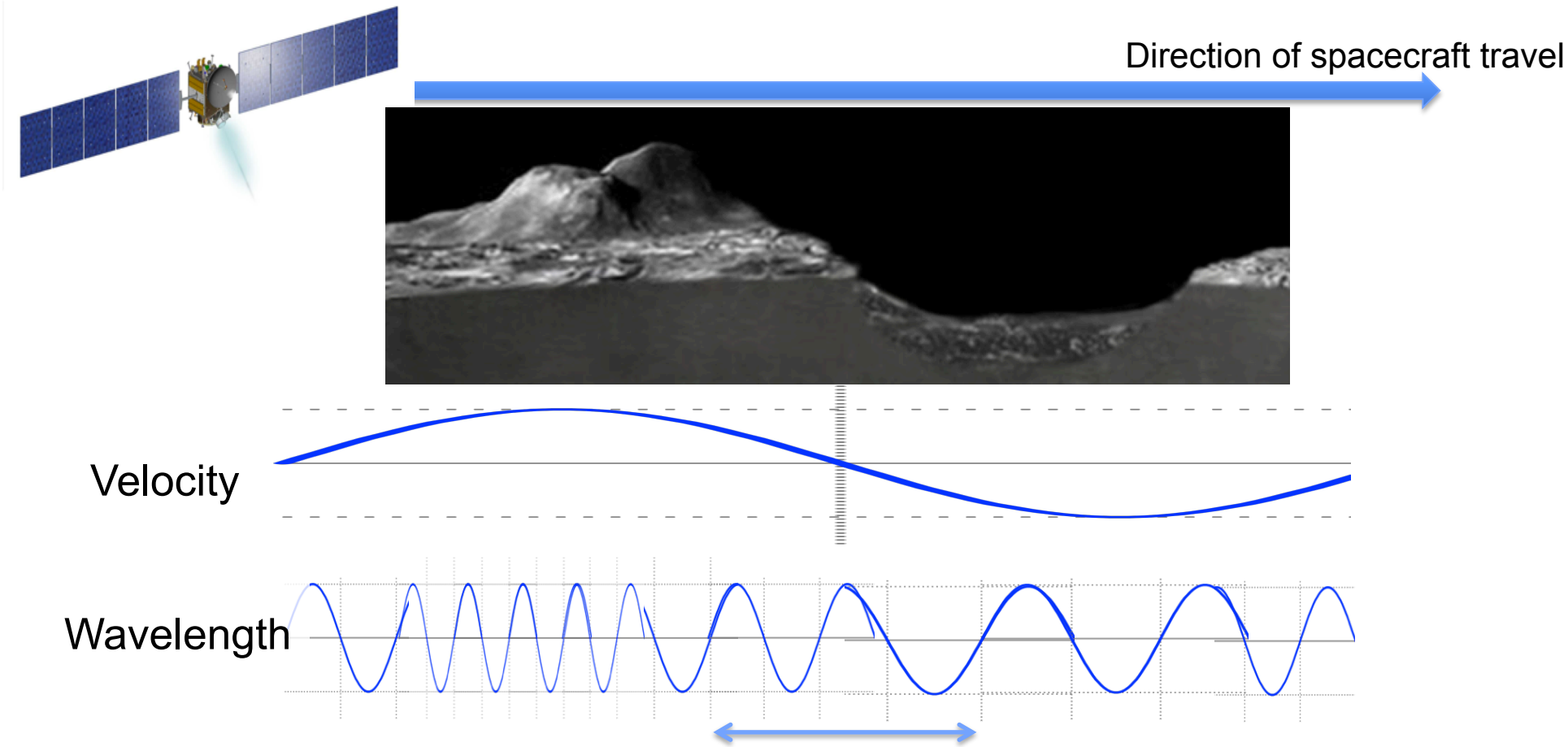


How to Observe Gravity



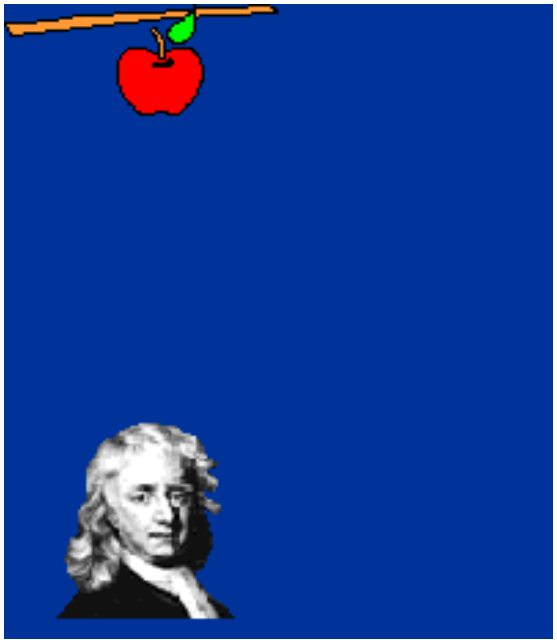
Spacecraft speeds up close to a dense object and slows down over a less dense one

Now Over A Planet

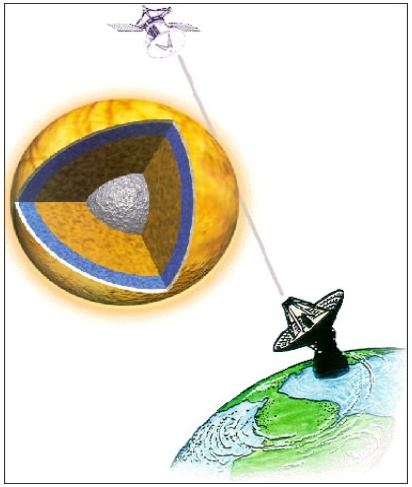


Spacecraft speeds up close to a mountain and slows down over a valley
(before accounting for sub-surface effects)

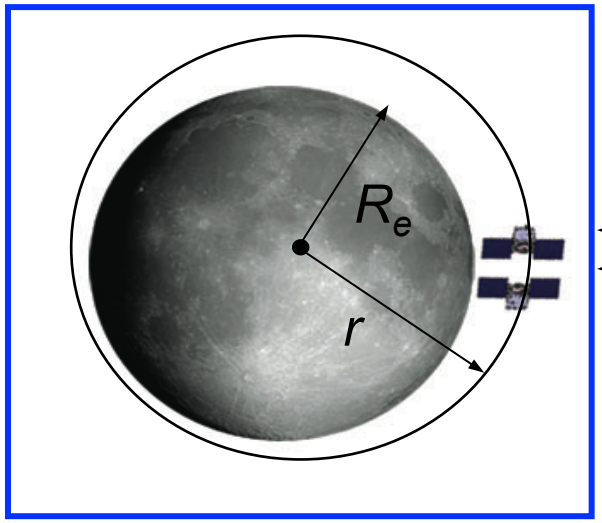
Gravity and Planetary Interiors



- Isaac Newton: Motion of satellite around a planet is largely due to the gravity field
- Deep space mission can investigate the interior structure of the target planetary body
- *Gravity Science* (branch of Radio Science) is a powerful tool for remote sensing the interior's density variations via precision Doppler tracking
- Classical method via a single spacecraft
  - Newer methods use two



Gravity from tracking data



lat: ø ,
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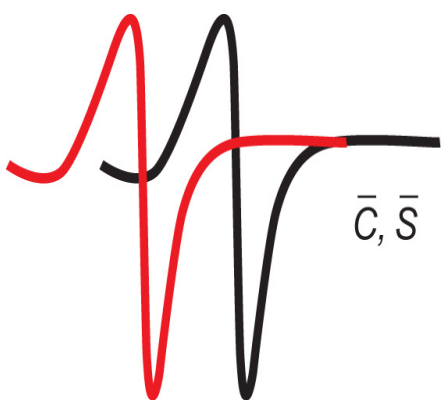
Deep Space Network

Potential function
Legendre polynomial
Spherical harmonic coefficients

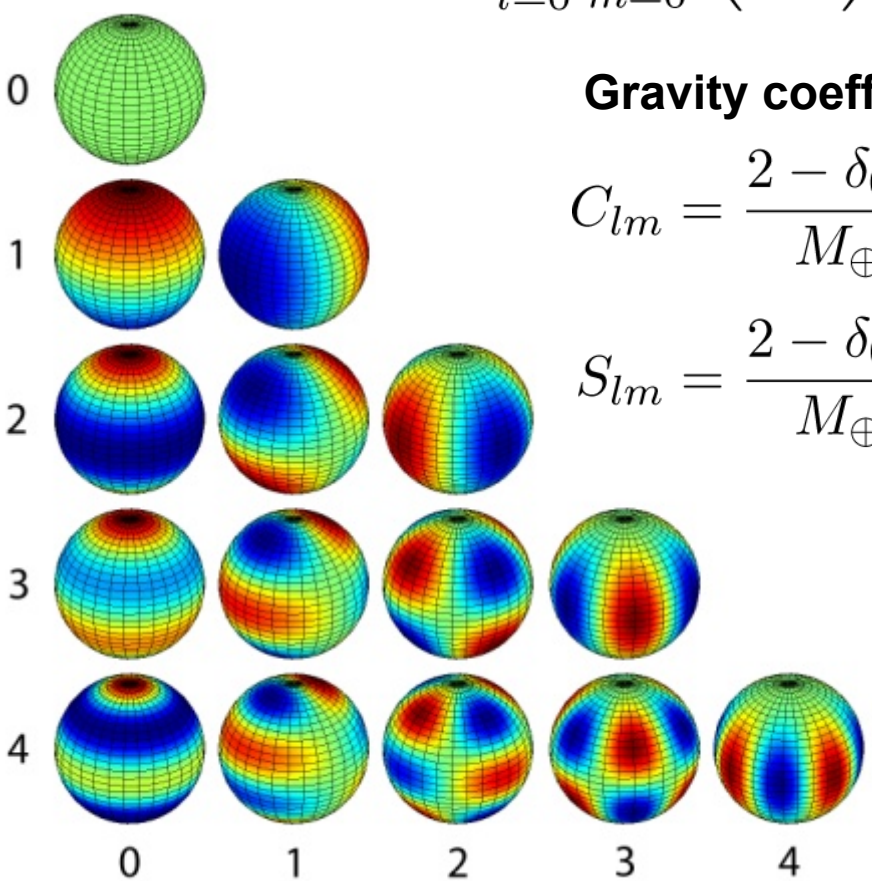


$$U(r, \phi, \lambda) = \frac{GM}{r} \sum_{n=0}^{180} \sum_{m=0}^{n} \left(\frac{R_e}{r}\right)^n \bar{P}_{nm}(\sin\phi)\left[\bar{C}_{nm}\cos(m\lambda) + \bar{S}_{nm}\sin(m\lambda)\right]$$

Gravity Field Representation

Potential function represented in terms of spherical harmonic expansion

$$V(r,\theta,\varphi) = \frac{GM_\oplus}{r}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\frac{R_\oplus^l}{r^l}\right)P_{lm}(\sin\theta)\left(C_{lm}\cos(m\varphi)+S_{lm}\sin(m\varphi)\right)$$



Gravity coefficients

$$C_{lm} = \frac{2-\delta_{0m}}{M_\oplus}\frac{(l-m)!}{(l+m)!}\int\frac{s^l}{R_\oplus^l}P_{lm}(\sin\theta)\cos(m\varphi)\rho(\boldsymbol{s})\mathrm{d}V$$

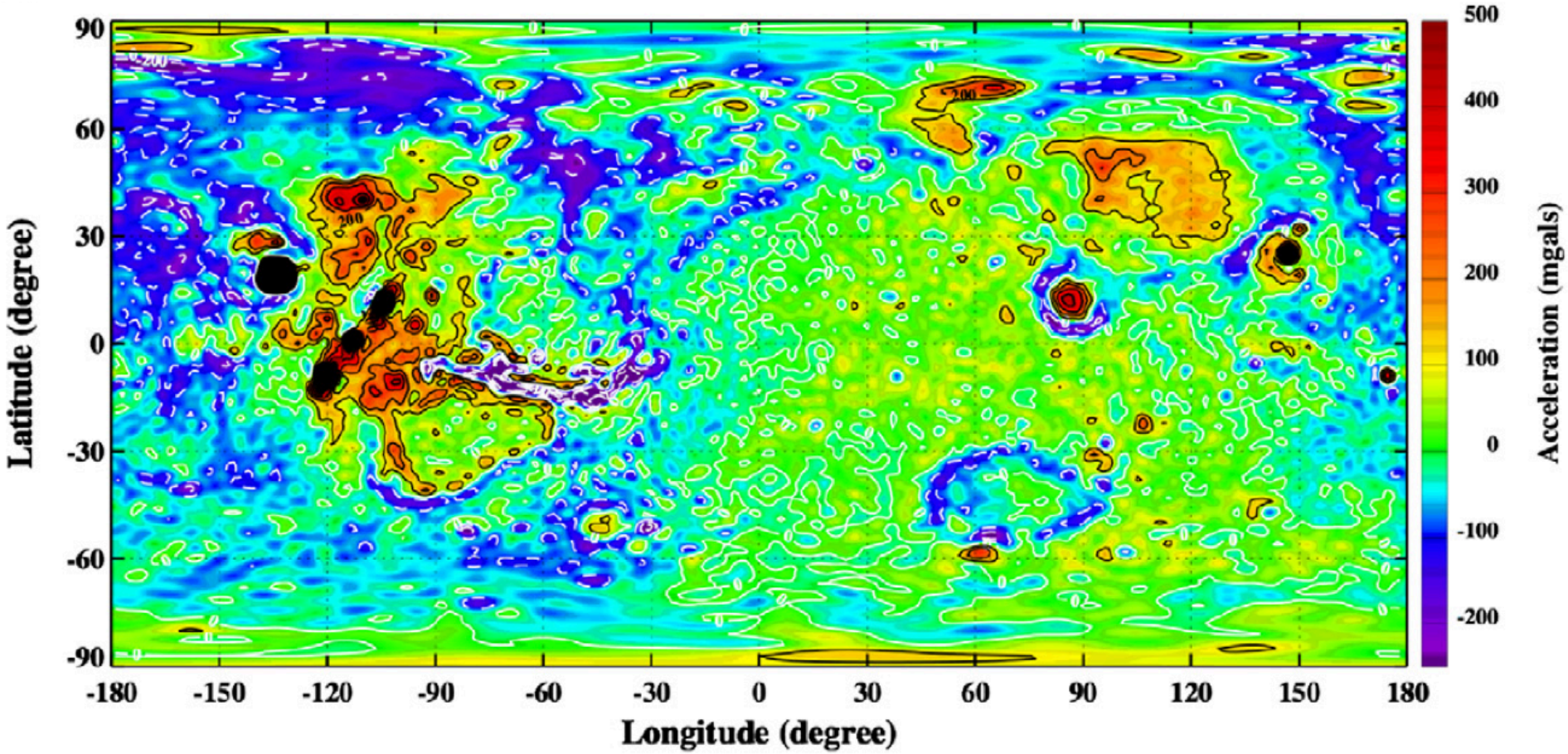
$$S_{lm} = \frac{2-\delta_{0m}}{M_\oplus}\frac{(l-m)!}{(l+m)!}\int\frac{s^l}{R_\oplus^l}P_{lm}(\sin\theta)\sin(m\varphi)\rho(\boldsymbol{s})\mathrm{d}V$$

Legendre Polynomials

$$P_{lm}(u) = \left(1-u^2\right)^{\frac{m}{2}}\frac{d^m}{du^m}P_l(u)$$

$$P_l(u) = \frac{1}{2^l l!}\frac{d^l}{du^l}\left(u^2-1\right)^l$$

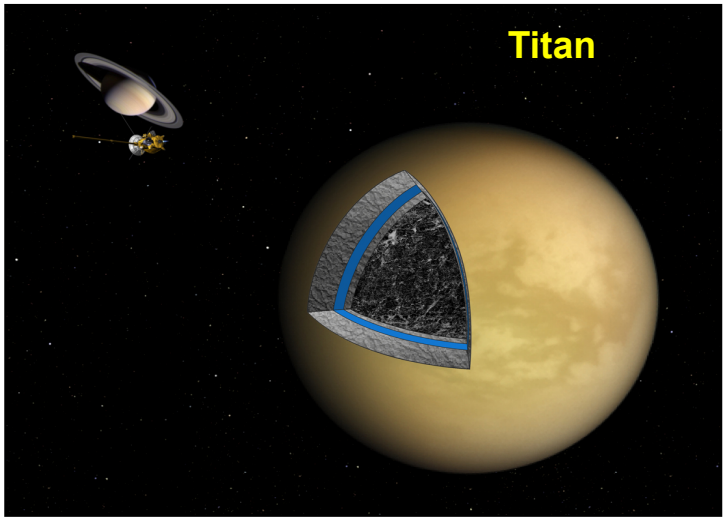
The Gravity Field of Mars

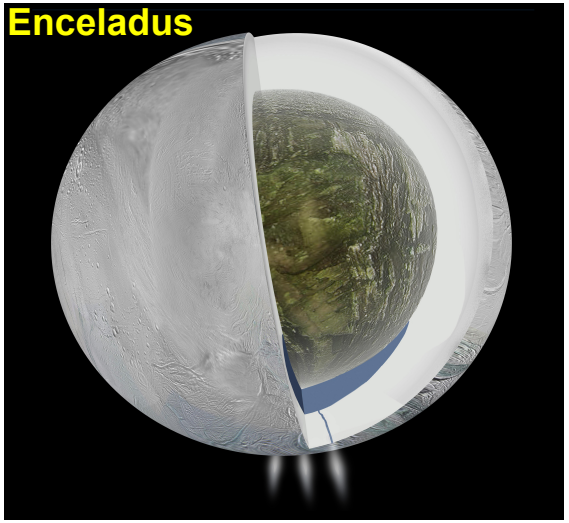


Surface gravity anomalies complete to degree and order 90 with respect to a reference ellipsoid (model MRO110B)

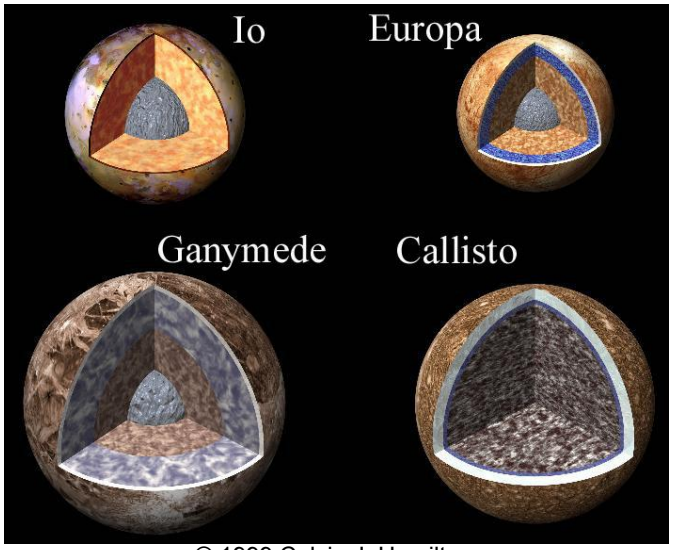
Konopliv et al., 2011

Moons of Large Planets





Models of the interiors of the Galilean satellites based on magnetic and gravity measurements

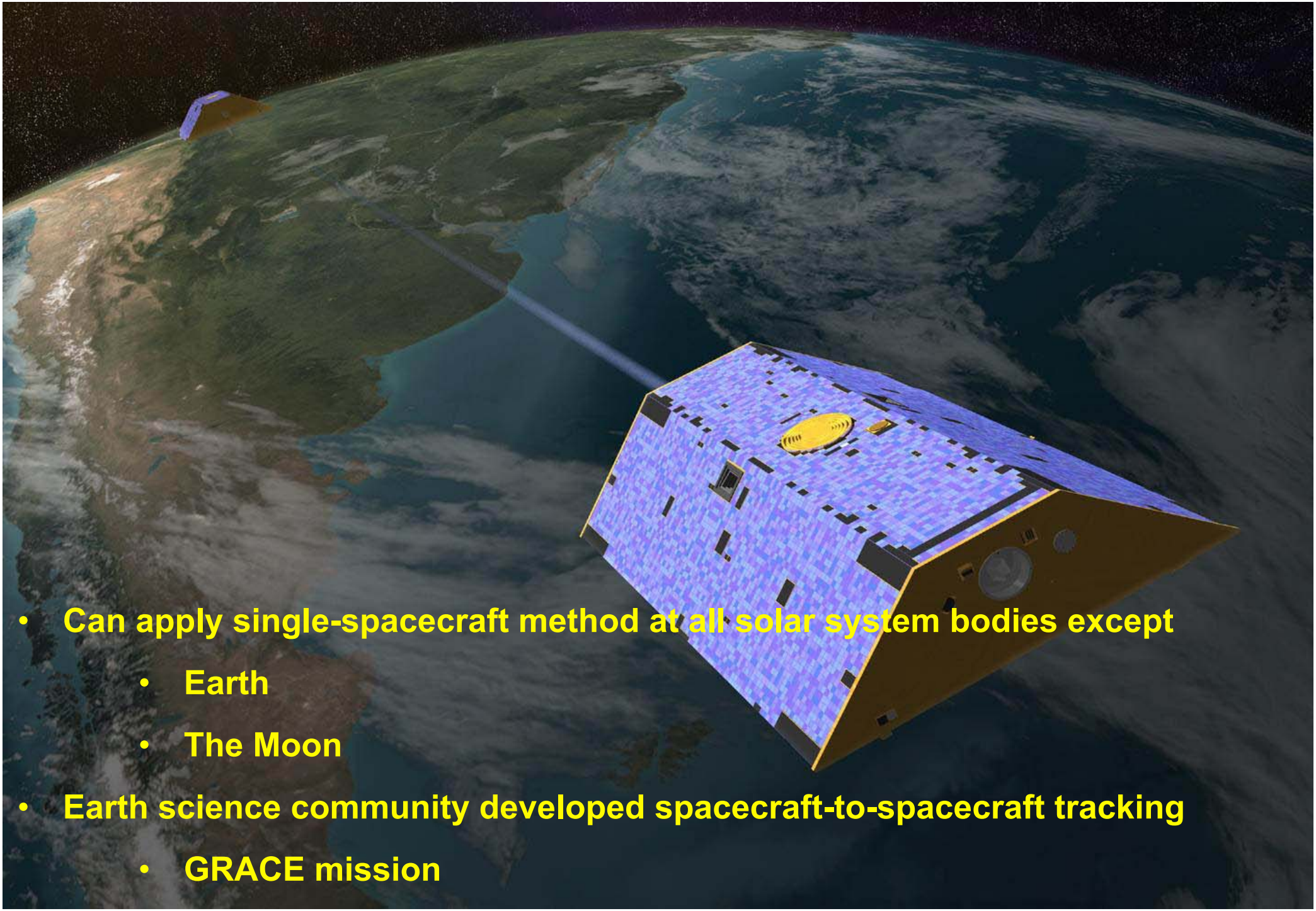


© 1999 Calvin J. Hamilton

Tidal observations by Cassini gravity team

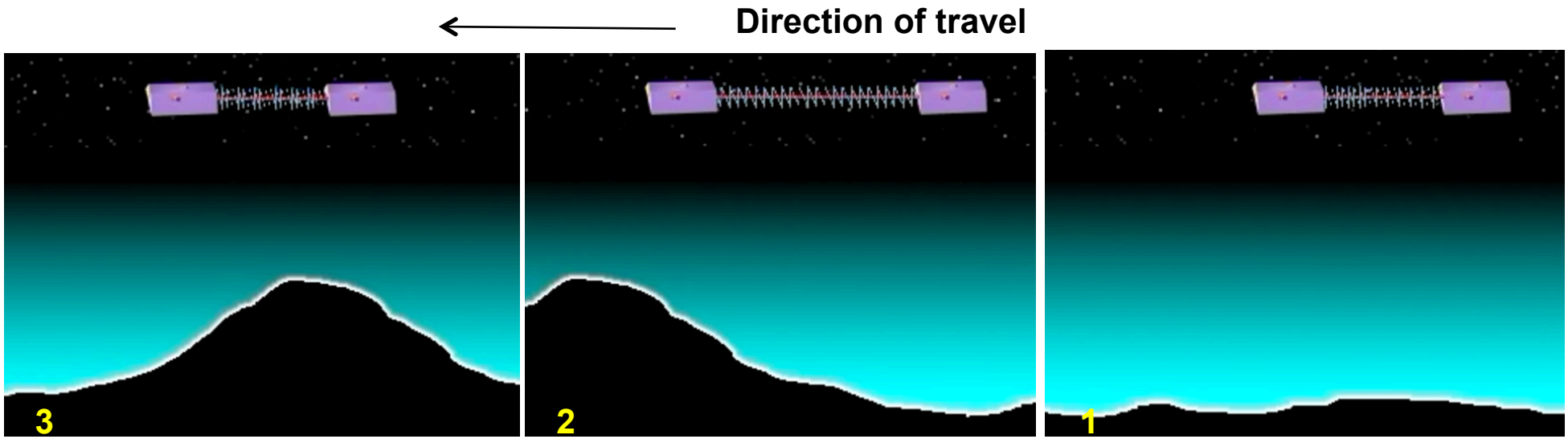
Titan: Iess et al., 2011 & 2012

Enceladus: Iess et al., 2014



- Can apply single-spacecraft method at all solar system bodies except
  - Earth
  - The Moon
- Earth science community developed spacecraft-to-spacecraft tracking
  - GRACE mission

GRACE Mission Concept



Separation distance between two spacecraft:

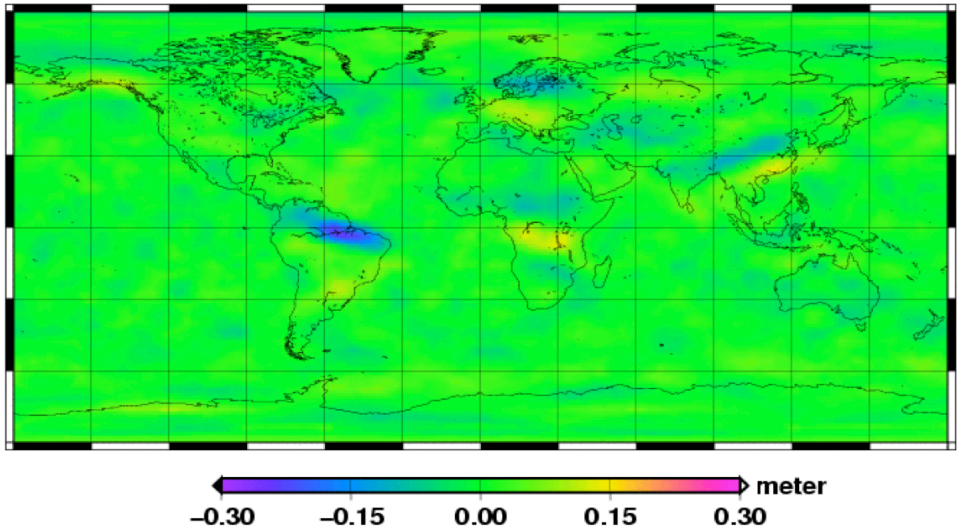
1. Nominal

2. Increases as leading spacecraft senses positive gravity anomaly due to mountain

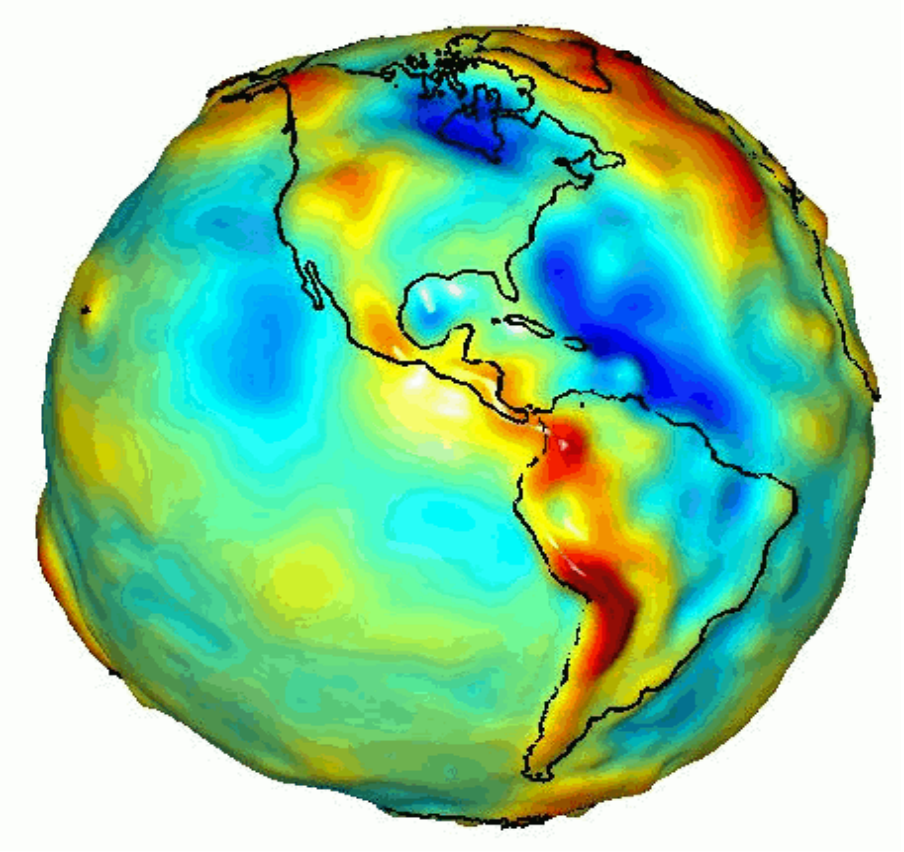
3. Decreases as trailing spacecraft senses positive gravity anomaly due to mountain

Earth's Gravity Varies with Time

- Earth's gravity varies due to mountains and valleys as well as different density in the materials beneath the surface
- Bumpiness changes monthly due to water movement



- Monthly surface mass variation in equivalent water height - annual wet & dry seasons
- Strongest signal over Amazon basin

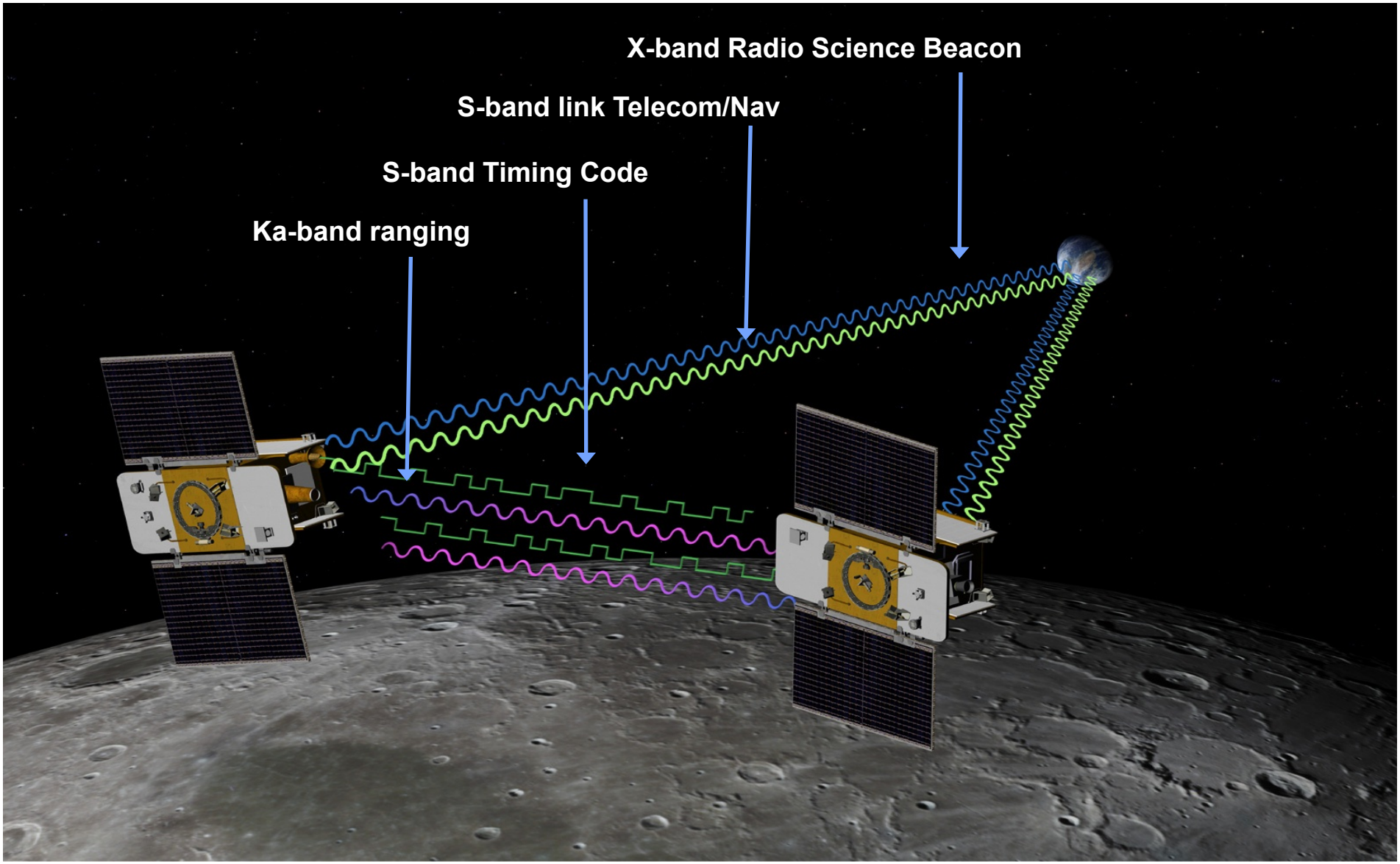


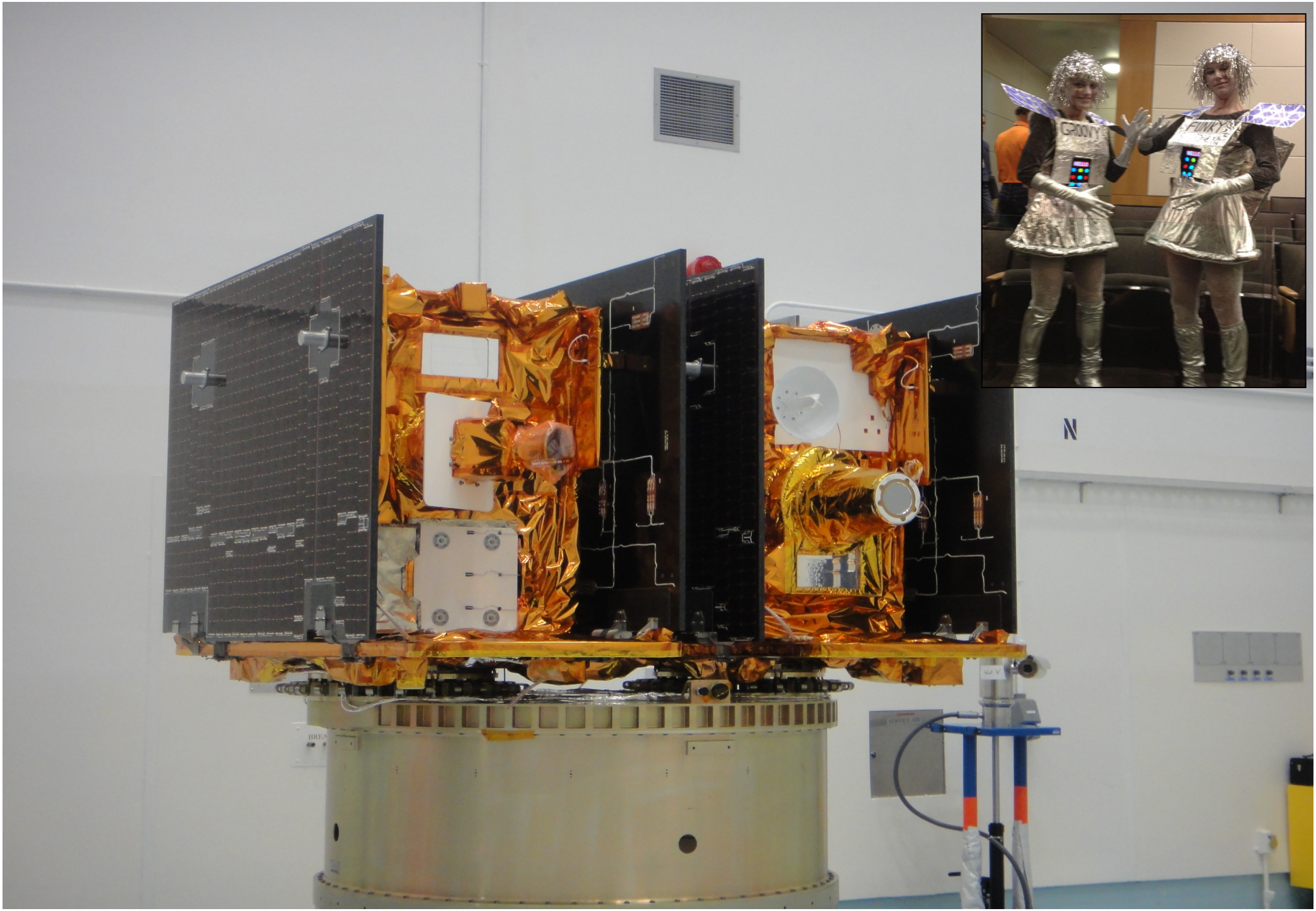
Case Study 1

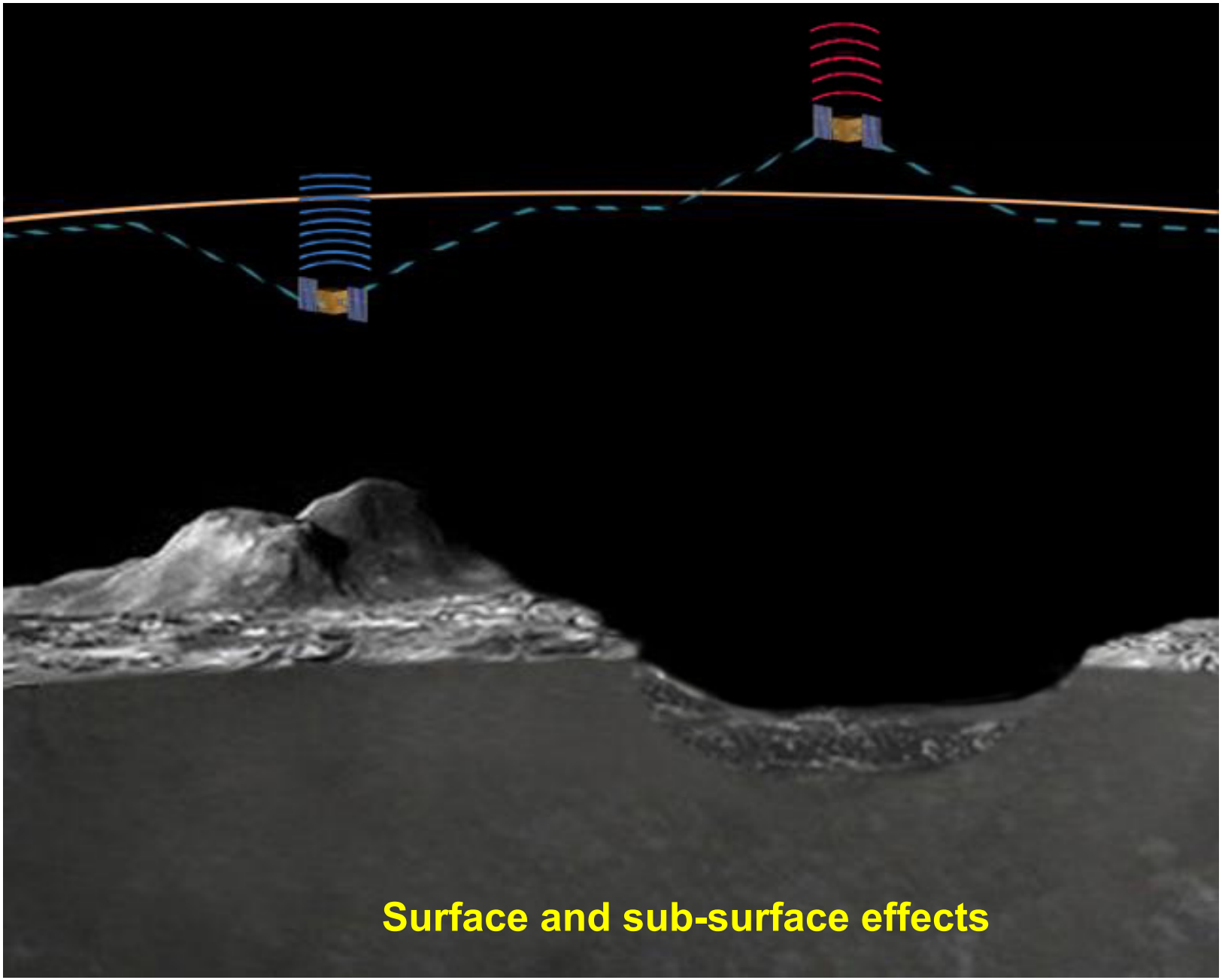
GRAVITY SCIENCE WITH GRAIL: *GRAVITY RECOVERY & INTERIOR LABORATORY*

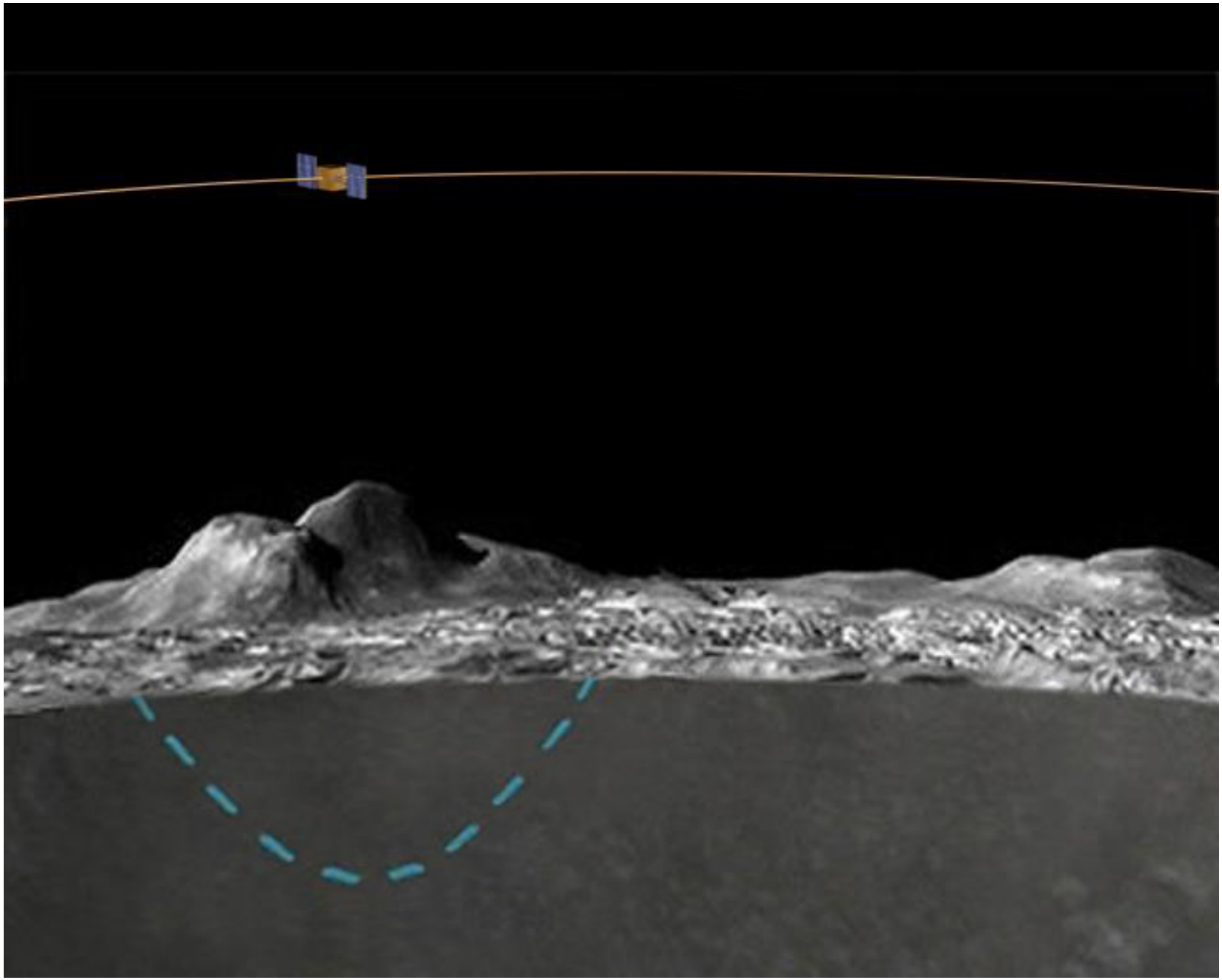
Sami Asmar

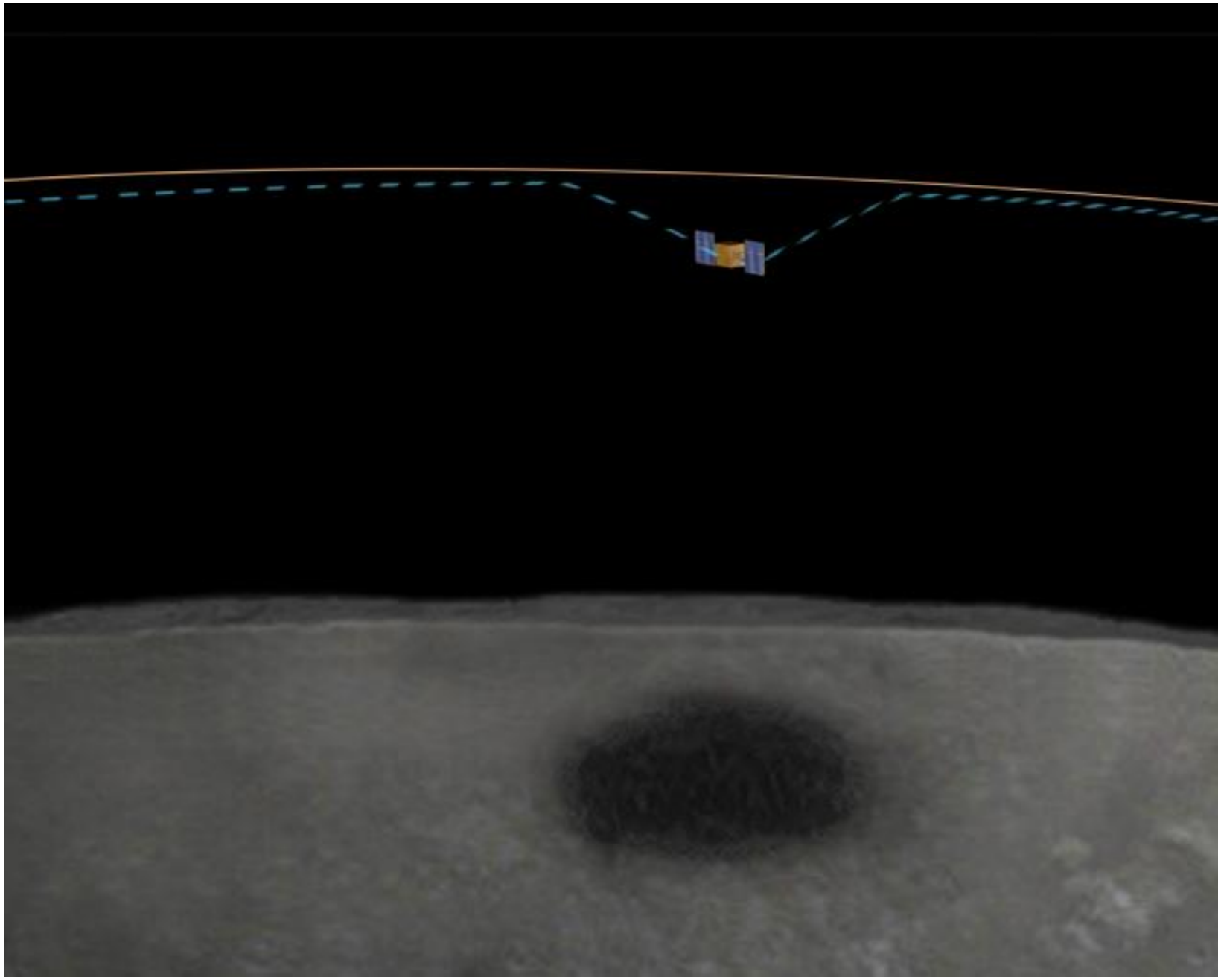
GRAIL: *Mission to the Moon*

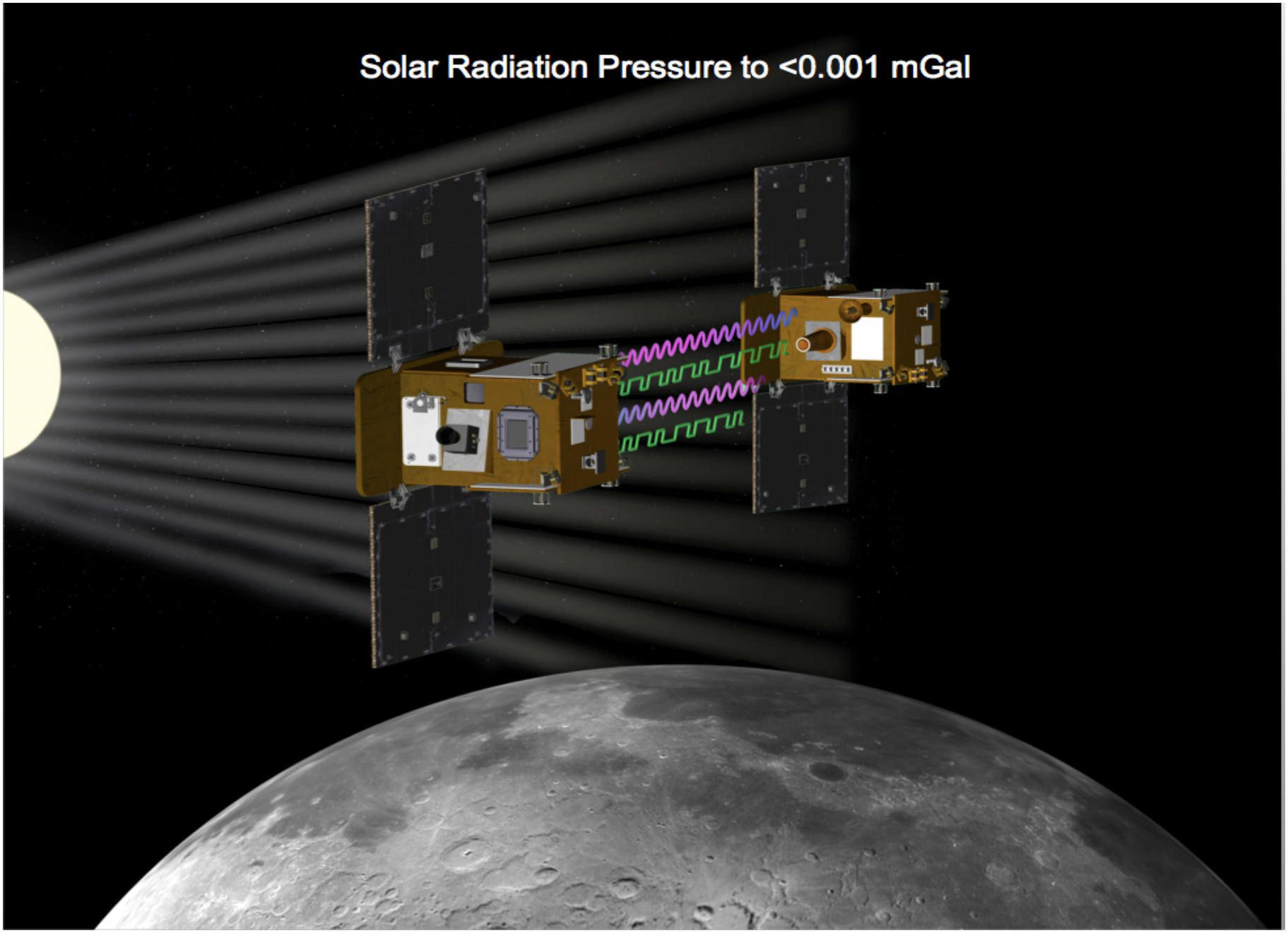



GROOVY
FUNKY

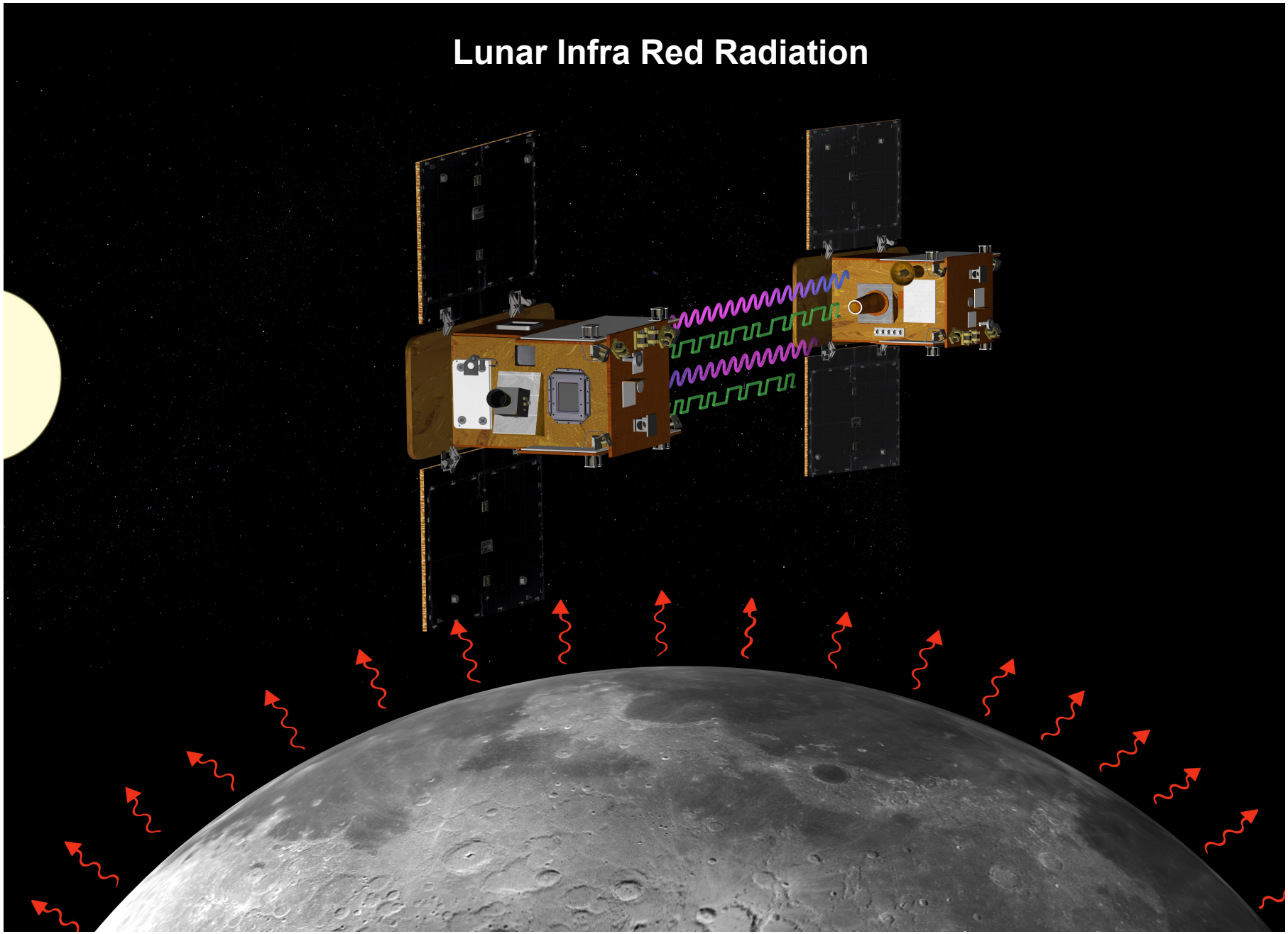

Surface and sub-surface effects



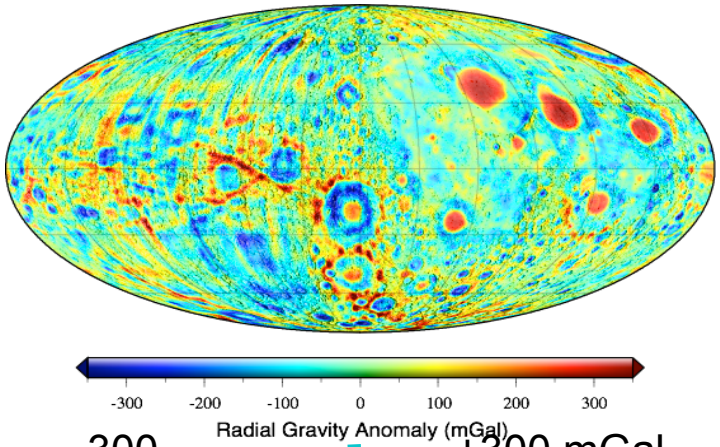



Solar Radiation Pressure to <0.001 mGal

Lunar Infra Red Radiation



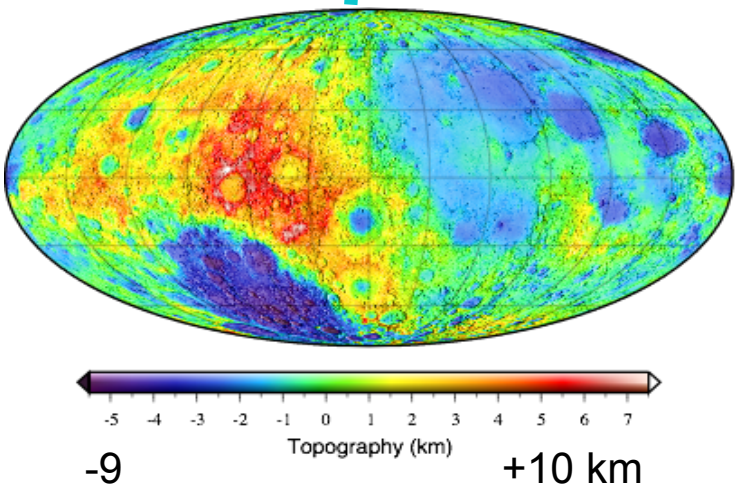
Gravity & Topography



gravity

-300 +300 mGal

geophysical model



topography

-9 +10 km

geochemistry
petrology
geology

crustal density

crustal thickness

elastic thickness

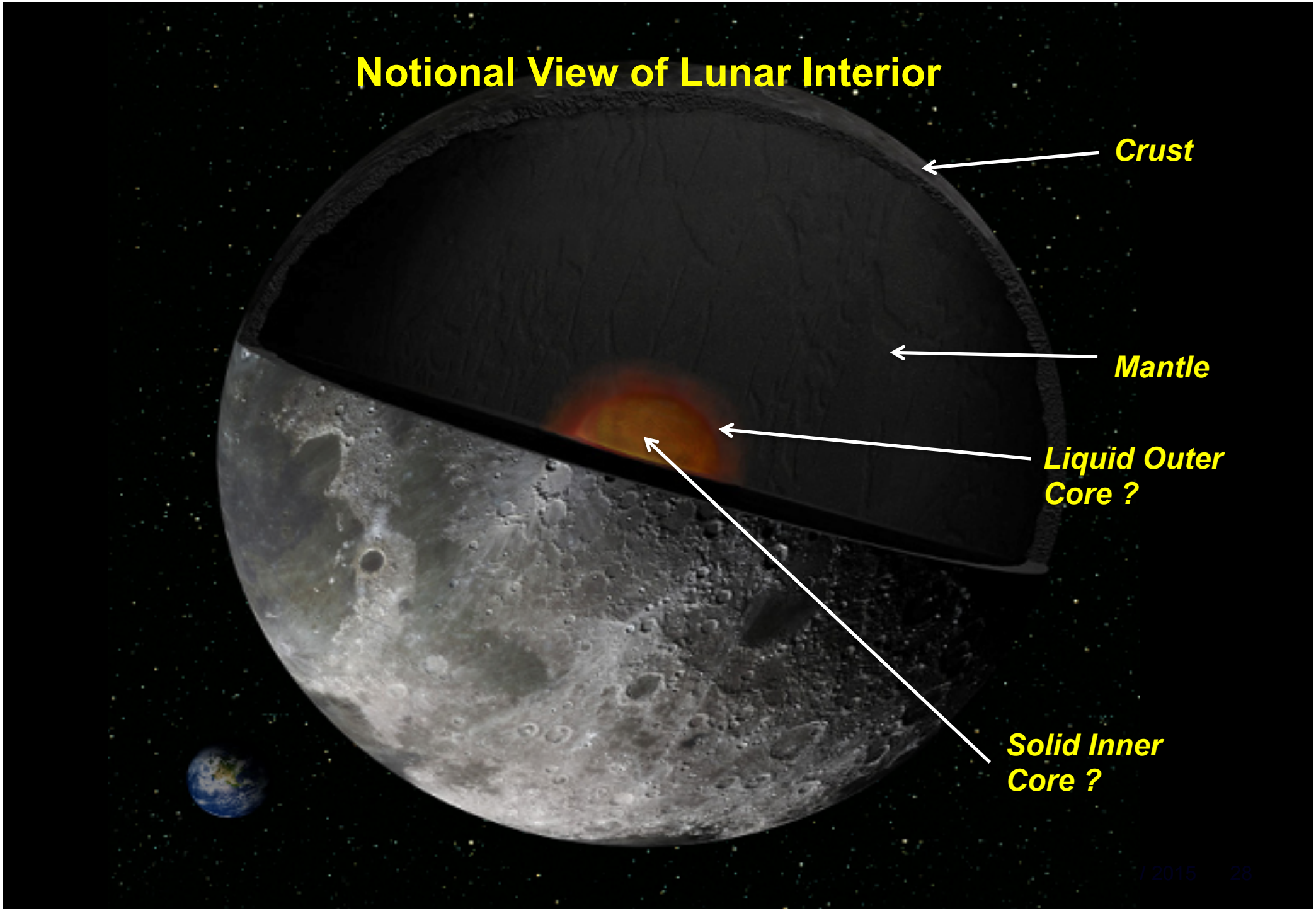
load density

surface-subsurface load ratio

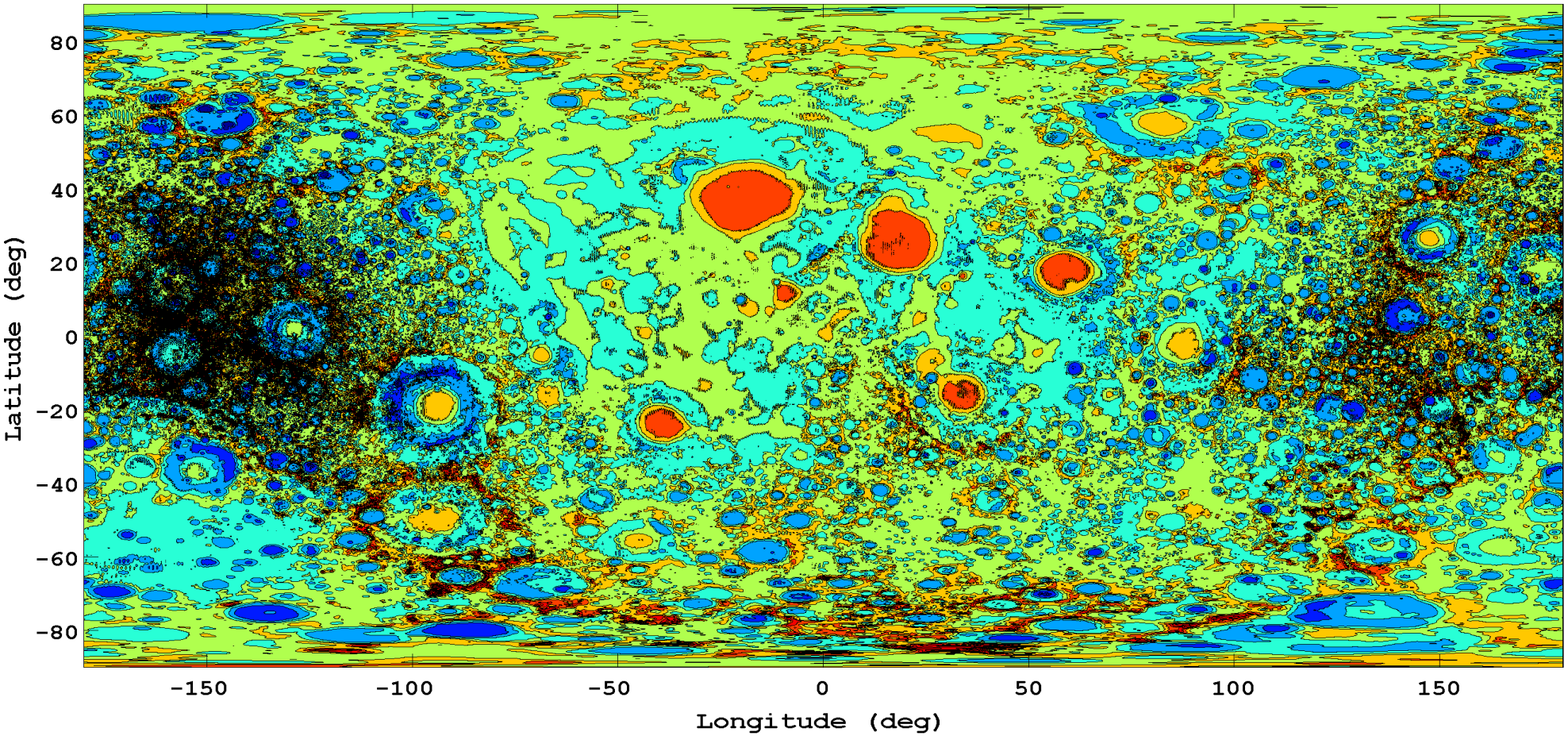
phase relationship of loads

internal structure
thermal evolution

Notional View of Lunar Interior



Lunar Gravity Map from GRAIL



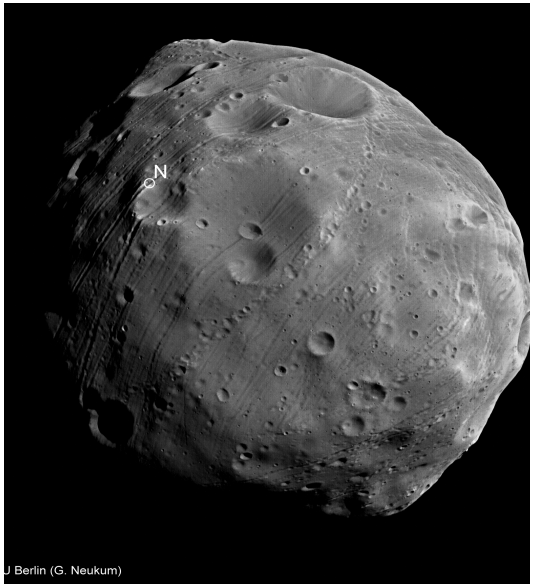
Konopliv et al., 2013 & 2014

Case Study 2

GRAVITY SCIENCE WITH MARS EXPRESS AT PHOBOS

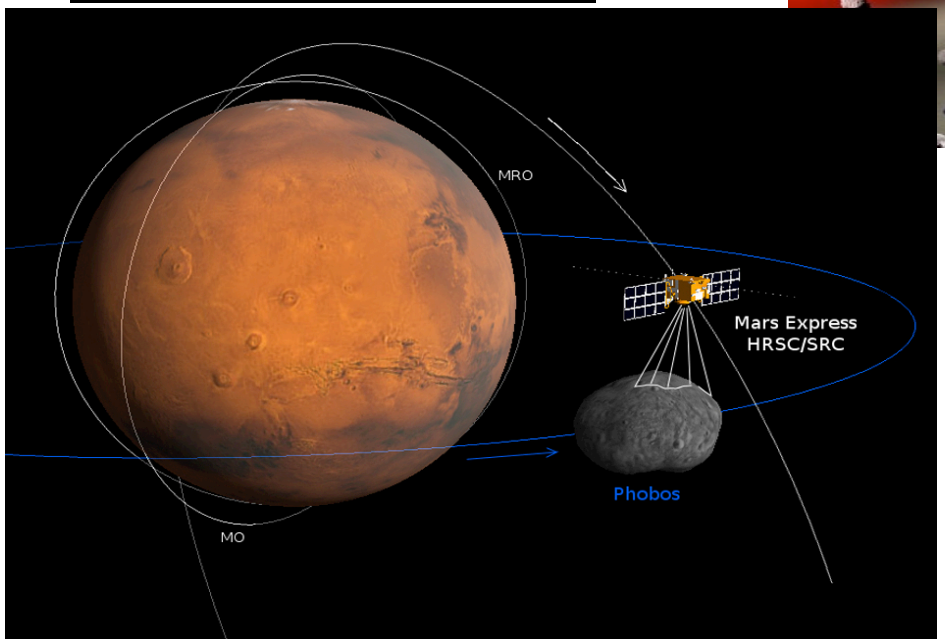
Tom Andert

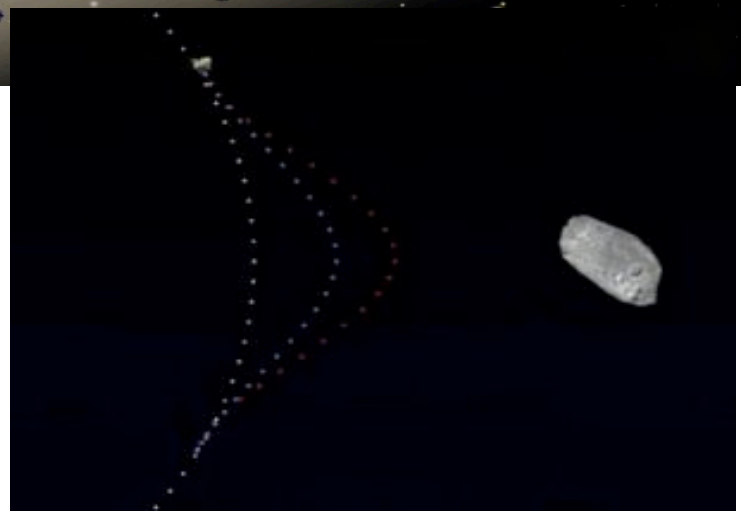
Phobos



- DSN-Mars Express radio science measurements
- Determined mass and bulk density

  $r_{bulk} = 1862 \pm 20\ kg/m^3$
- Bulk density lower than solid bodies
- High porosity
- Phobos re-accreted in place!(?)





Special Spherical Harmonic Coefficients

- Gravity coefficients of degree 1 defines the offset between the coordinate system and the center of the mass of the body

$$C_{10} = \Delta z / R_0 \quad \Rightarrow \Delta z = C_{10} \cdot R_0$$

$$C_{11} = \Delta x / R_0 \quad \Rightarrow \Delta x = C_{11} \cdot R_0$$

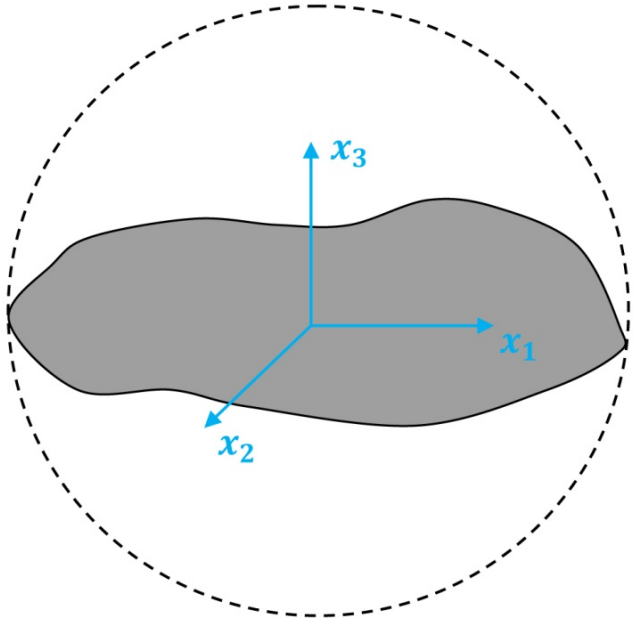
$$S_{11} = \Delta y / R_0 \quad \Rightarrow \Delta y = S_{11} \cdot R_0$$

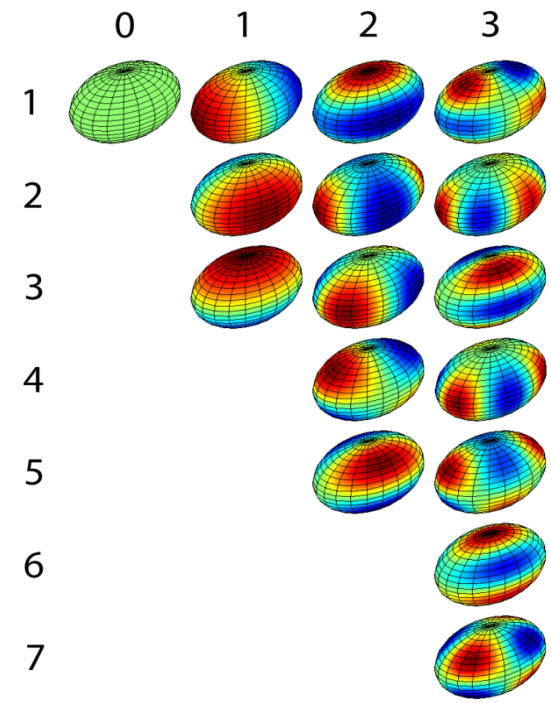
- C_{21} and S_{21} vanish as long as the z-axis is aligned with the main axis of inertia

$\Rightarrow$ Some coefficients vanish as a consequence of an appropriate choice of the coordinate system

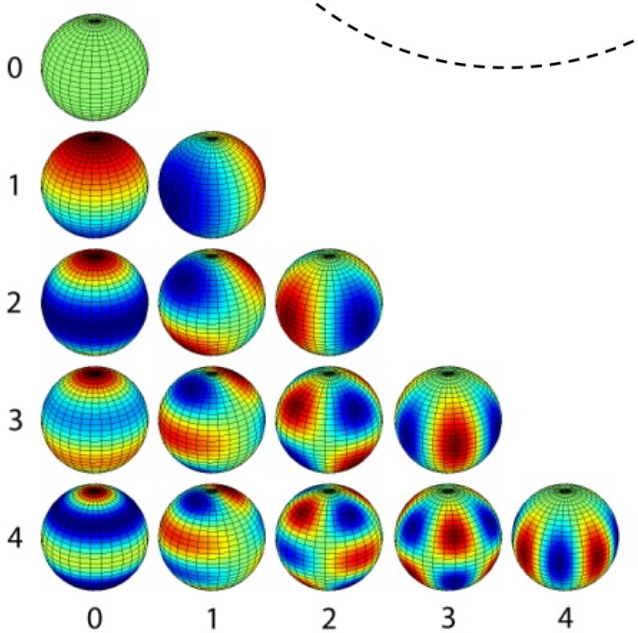
- All S_{n0} are zero by definition
- C_{20} represents the flattening of the body

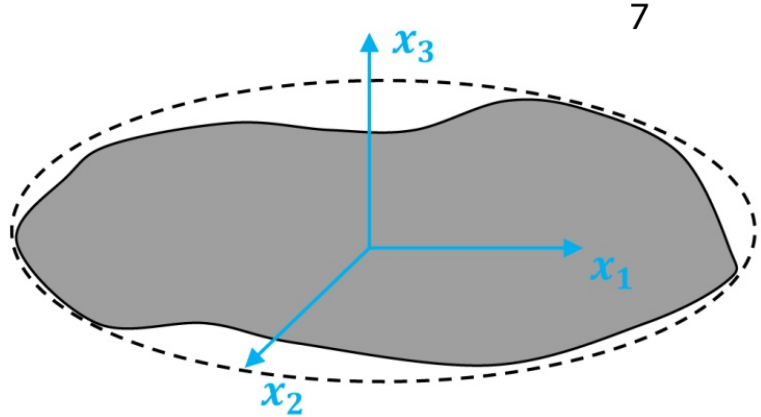
Spherical vs. Ellipsoidal Coordinates



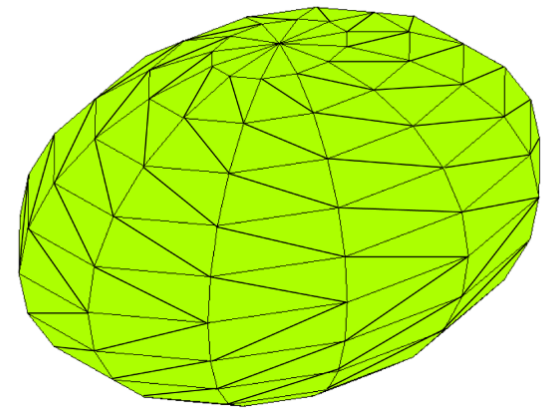


vs.





Gravity Potential in Ellipsoidal Coordinates

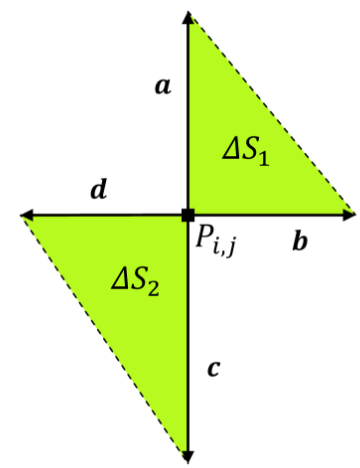


$$V(\lambda_1,\lambda_2,\lambda_3) = GM_\oplus \sum_{n=0}^{\infty}\sum_{p=1}^{2n+1} \alpha_n^p \frac{E_n^p(\lambda_1)}{E_n^p(\lambda_1^{\mathrm{ref}})} E_n^p(\lambda_2) E_n^p(\lambda_3)$$

$$\text{für } \lambda_1 \le \lambda_1^{\mathrm{ref}}$$

$$V(\lambda_1,\lambda_2,\lambda_3) = GM_\oplus \sum_{n=0}^{\infty}\sum_{p=1}^{2n+1} \alpha_n^p \frac{F_n^p(\lambda_1)}{F_n^p(\lambda_1^{\mathrm{ref}})} E_n^p(\lambda_2) E_n^p(\lambda_3)$$

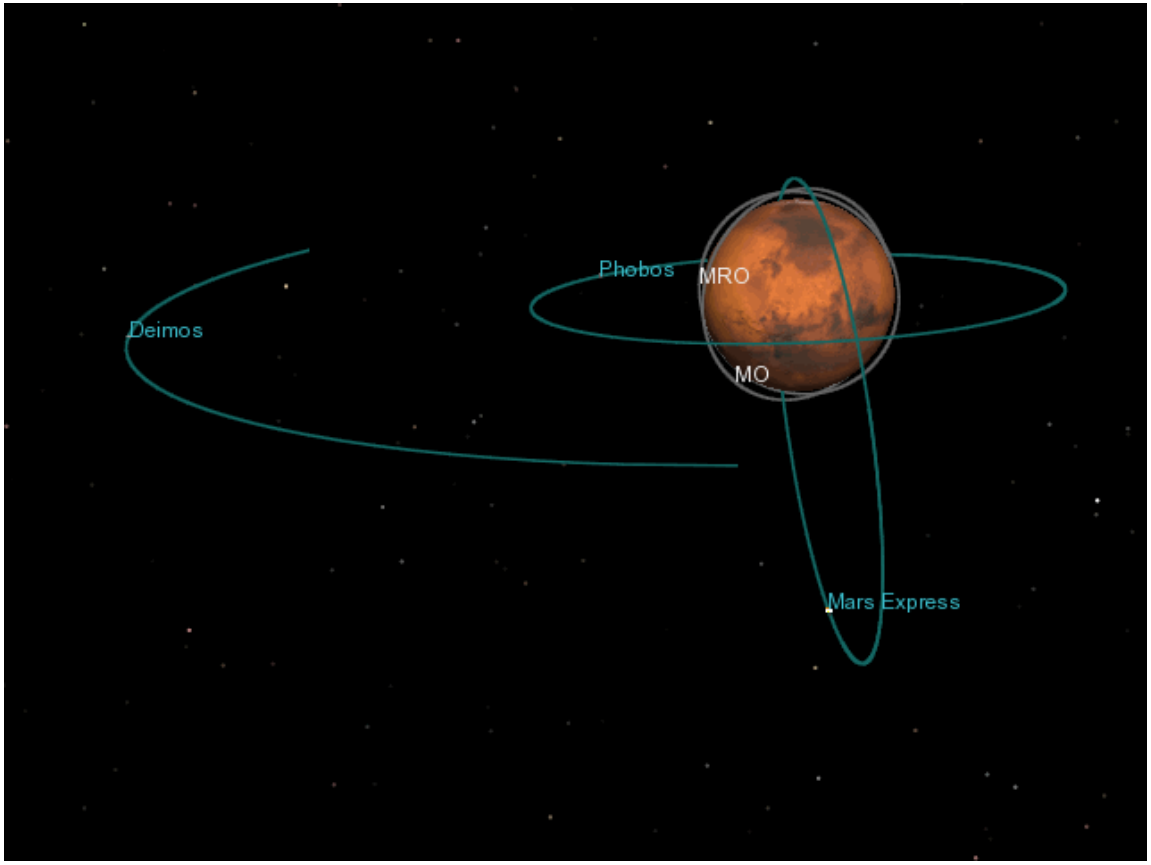
$$\text{für } \lambda_1 \ge \lambda_1^{\mathrm{ref}}$$



$$\alpha_n^p = \frac{1}{\gamma_n^p} \sum_{i=1}^{2NM} \frac{V(x_{1i},x_{2i},x_{3i})}{GM_\oplus} \frac{\Psi_n^p(x_{1i},x_{2i},x_{3i}) P_n^p(\lambda_{2i}) P_n^p(\lambda_{3i}) \Delta S_i}{\Psi_n^p(\lambda_{1i}) \sqrt{\left(\lambda_{1i}^2-\lambda_{2i}^2\right)\left(\lambda_{1i}^2-\lambda_{3i}^2\right)}}$$

Phobos Gravity Measurement

Mars Express is the first spacecraft since nearly 20 years which is able to perform close flybys at the Mars moon Phobos. So far, four flybys were performed:



http://www.esa.int/SPECIALS/Mars_Express/

- 23.03.2006 at **460** km

  ⇒ GM estimate

- 17.07.2008 at **275** km

  ⇒ GM estimate

(*Andert et al., 2010*)

- 03.03.2010 at **77** km

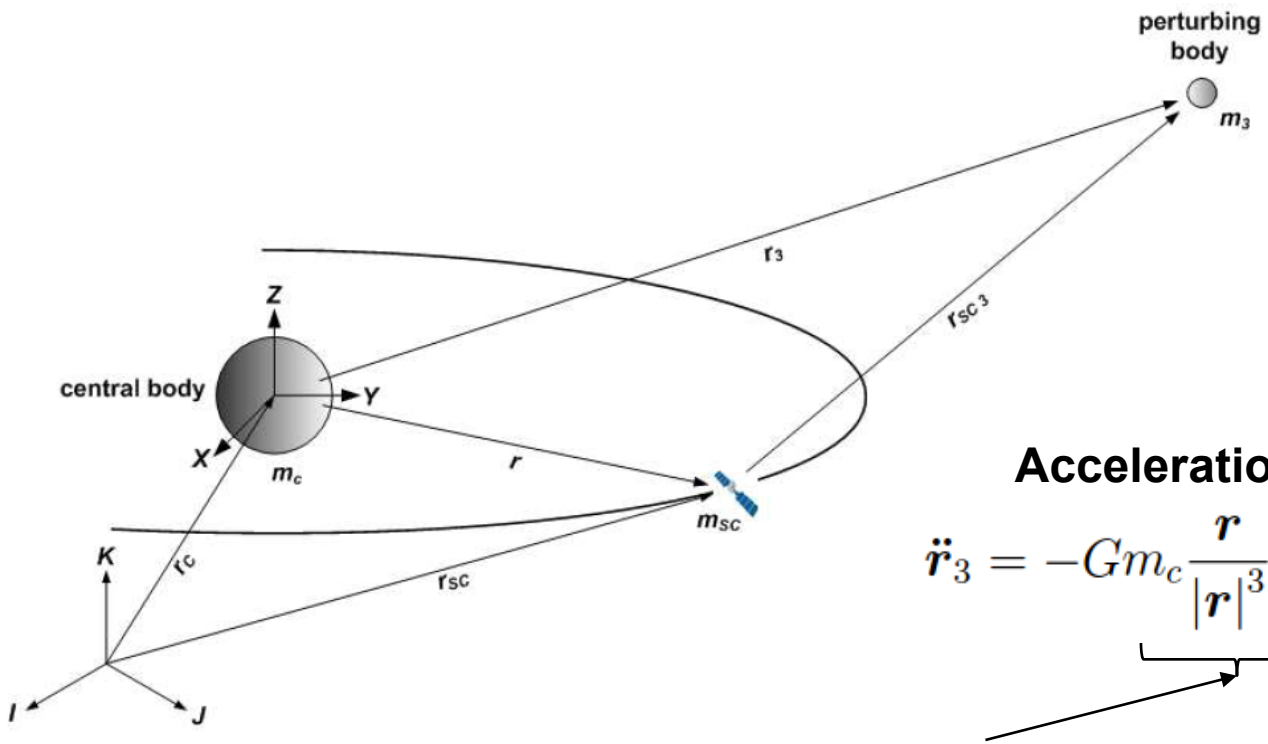
  ⇒ C_{20} estimate

(*Pätzold and Andert et al., 2014*)

- 29.12.2013 at **58** km

  ⇒ C_{20} and C_{22} estimate

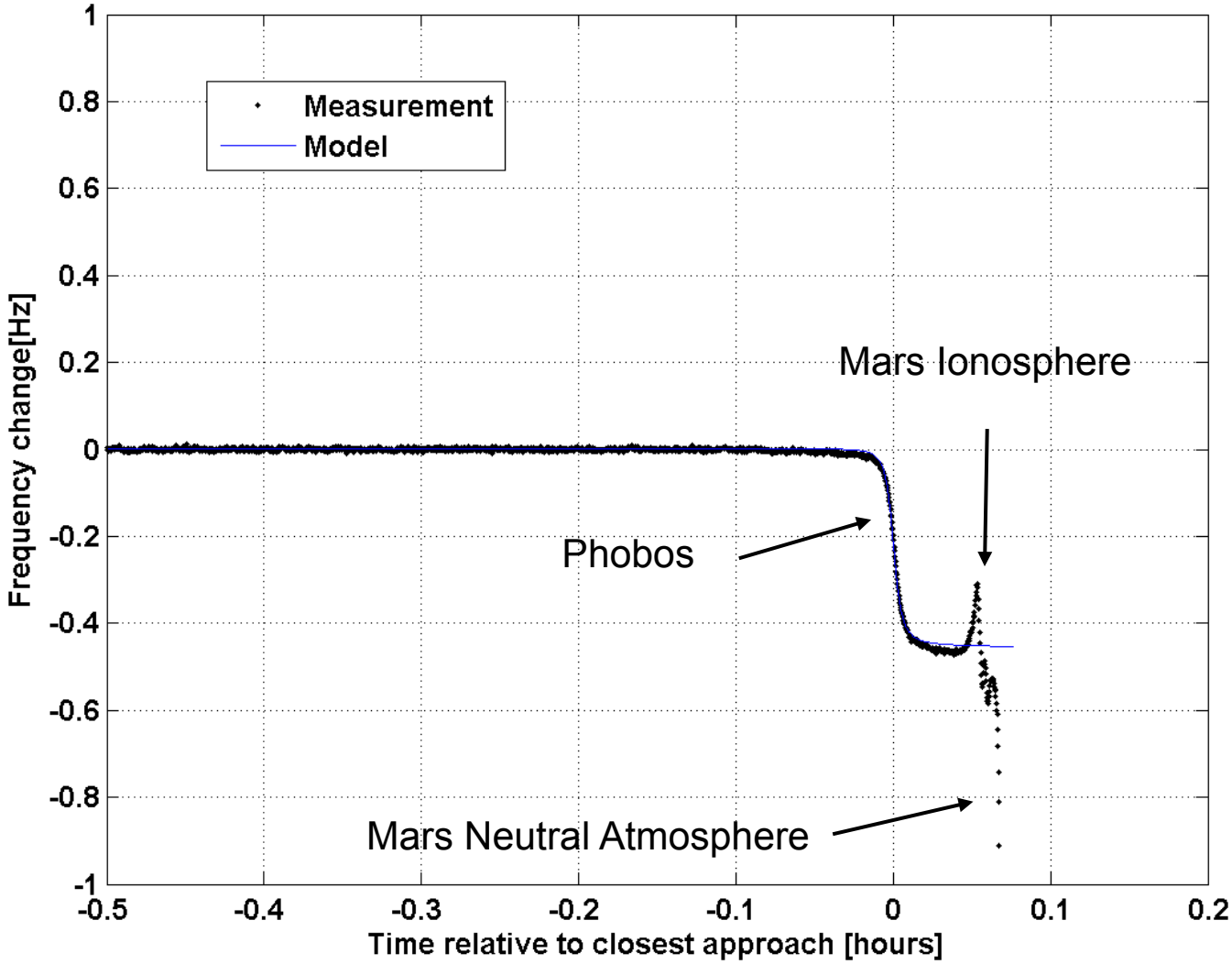
Third Body Perturbations



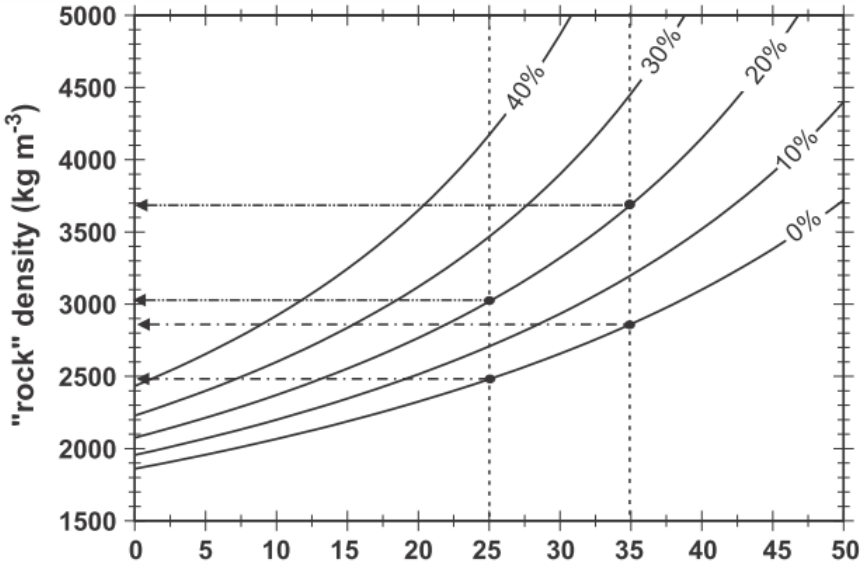
Acceleration on the SC

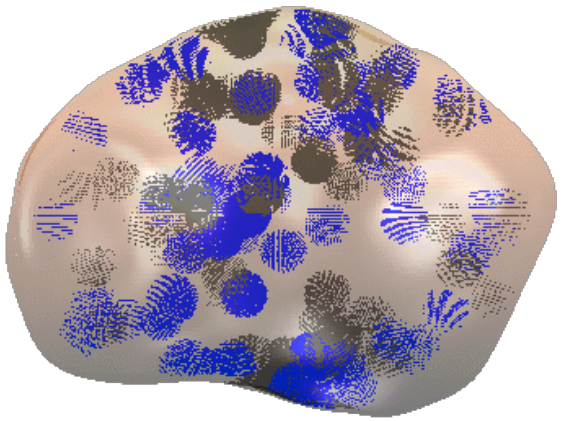
$$\ddot{\boldsymbol{r}}_3 = \underbrace{-Gm_c\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}}_{\text{Acceleration by the central body, for example Mars gravity field (n = 110)}} \underbrace{- Gm_3\left(\frac{\boldsymbol{r}-\boldsymbol{r}_3}{|\boldsymbol{r}-\boldsymbol{r}_3|^3} + \frac{\boldsymbol{r}_3}{|\boldsymbol{r}_3|^3}\right)}_{\text{Acceleration caused by the third body, for example Phobos}}$$

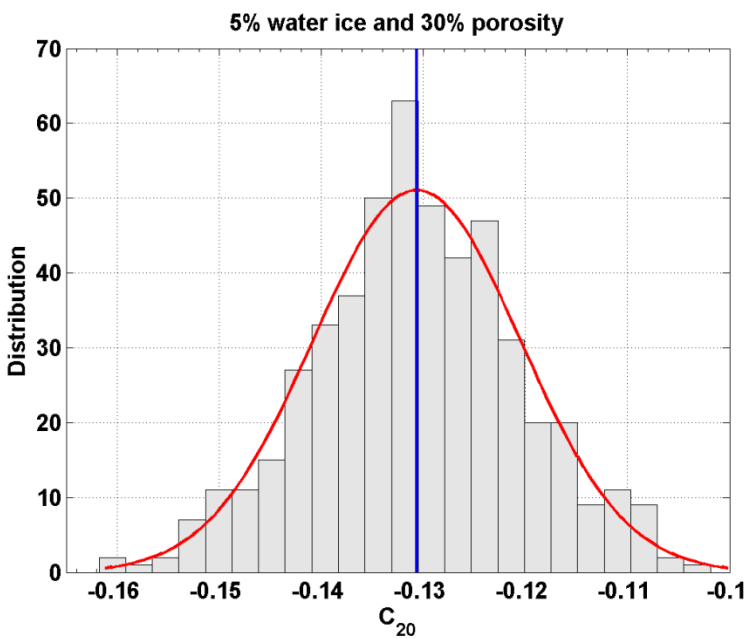
Phobos Flyby, Dec. 2013, @ 58 km

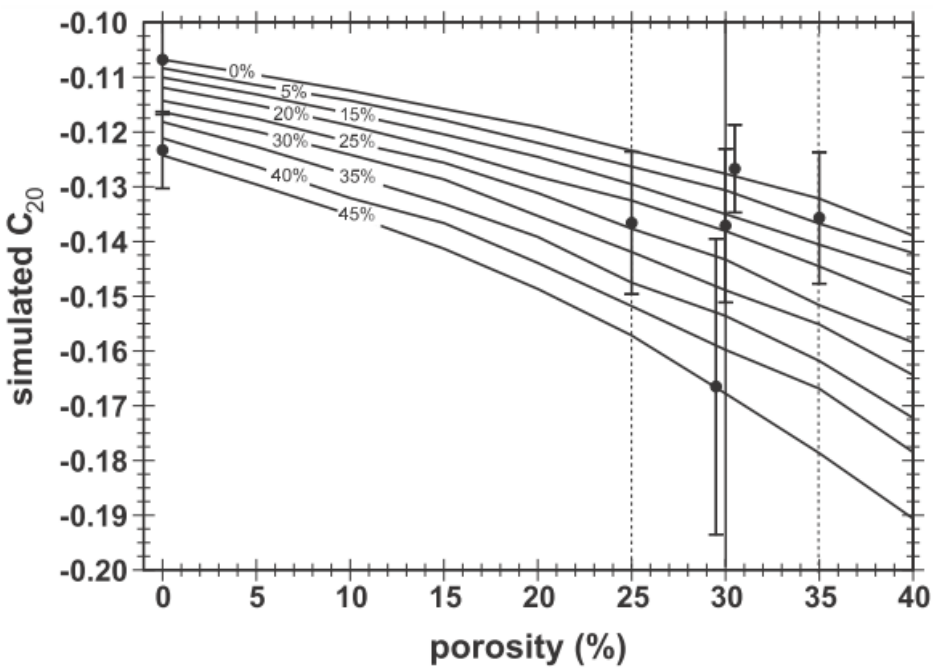


Porosity and Water ice content from gravity









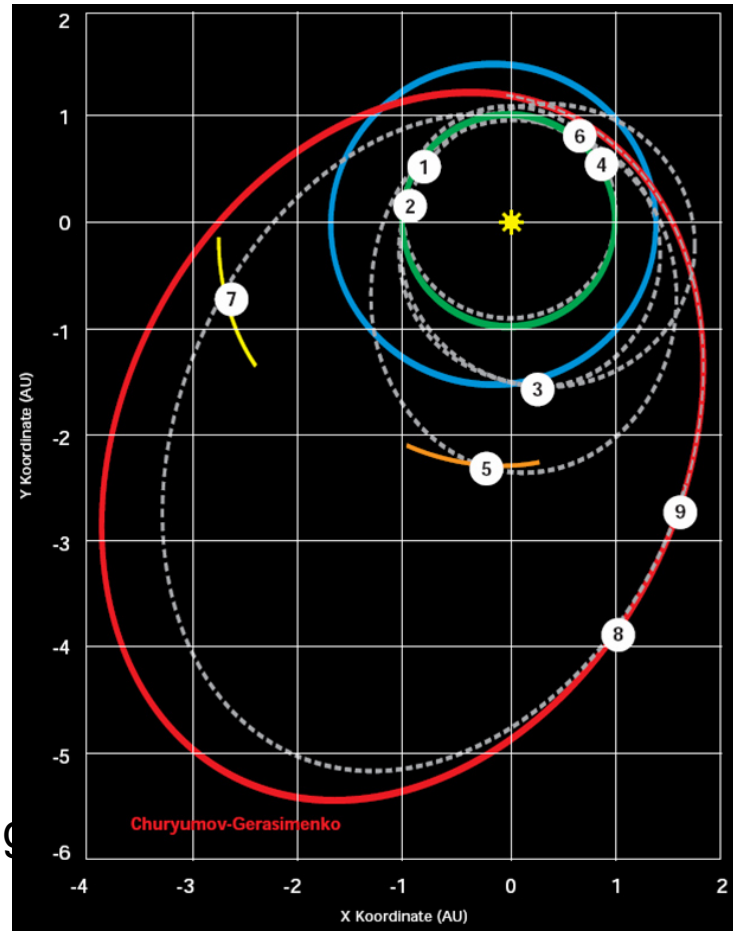
Case Study 3

GRAVITY SCIENCE WITH ROSETTA

Tom Andert

Rosetta's Journey

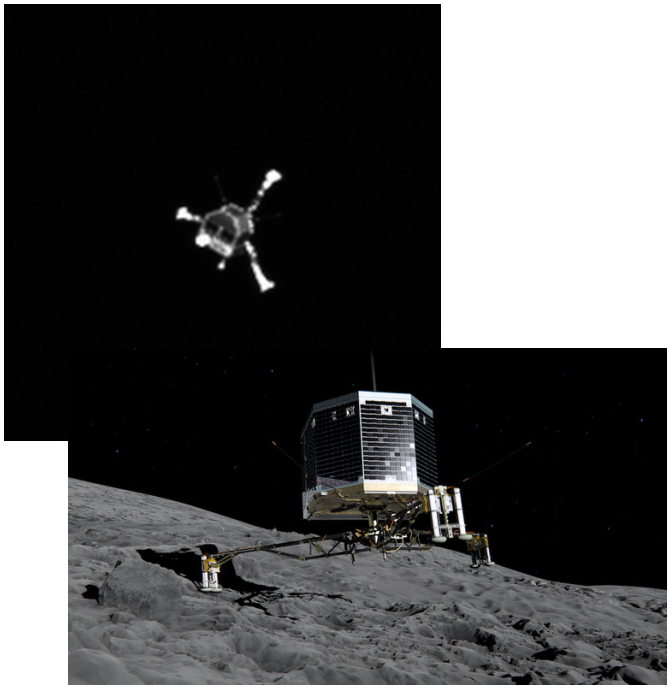
- Launch of Rosetta to comet 67P/Churyumov-Gerasimenko: 2 March 2004 (1)
- 1st Earth gravity assist: 4 March 2005 (2)
- Mars gravity assist: 25 February 2007 (3)
- 2nd Earth gravity assist: 13 November 2007 (4)
- Asteroid Steins flyby: 5 September 2008 (5)
- 3rd Earth gravity assist: 13 November 2009 (6)
- Asteroid Lutetia flyby: 10 July 2010 (7)
- Enter deep space hibernation: 8 June 2011
- Exit deep space hibernation: 20 January 2014
- Major comet rendezvous manoeuvre: May 2014 (8)
- Arrive at comet: 6 August 2014
- Philae lander delivery: 12 November 2014 (9)
- Perihelion passage (Closest approach to Sun): 13 Aug
- Mission end: 31 December 2015

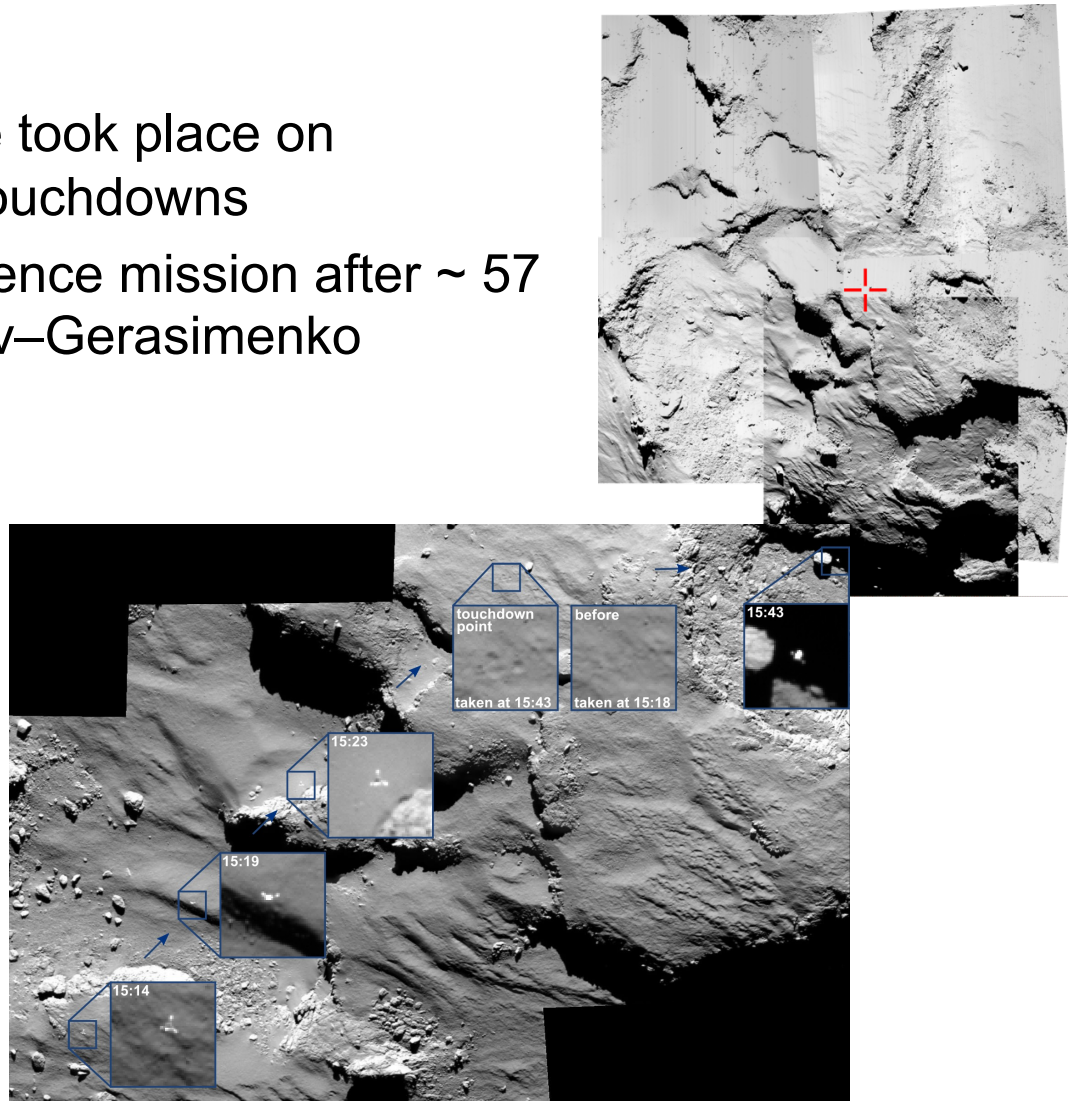


Phillae Landing

The successful landing of Philae took place on 12 November 2014 after three touchdowns

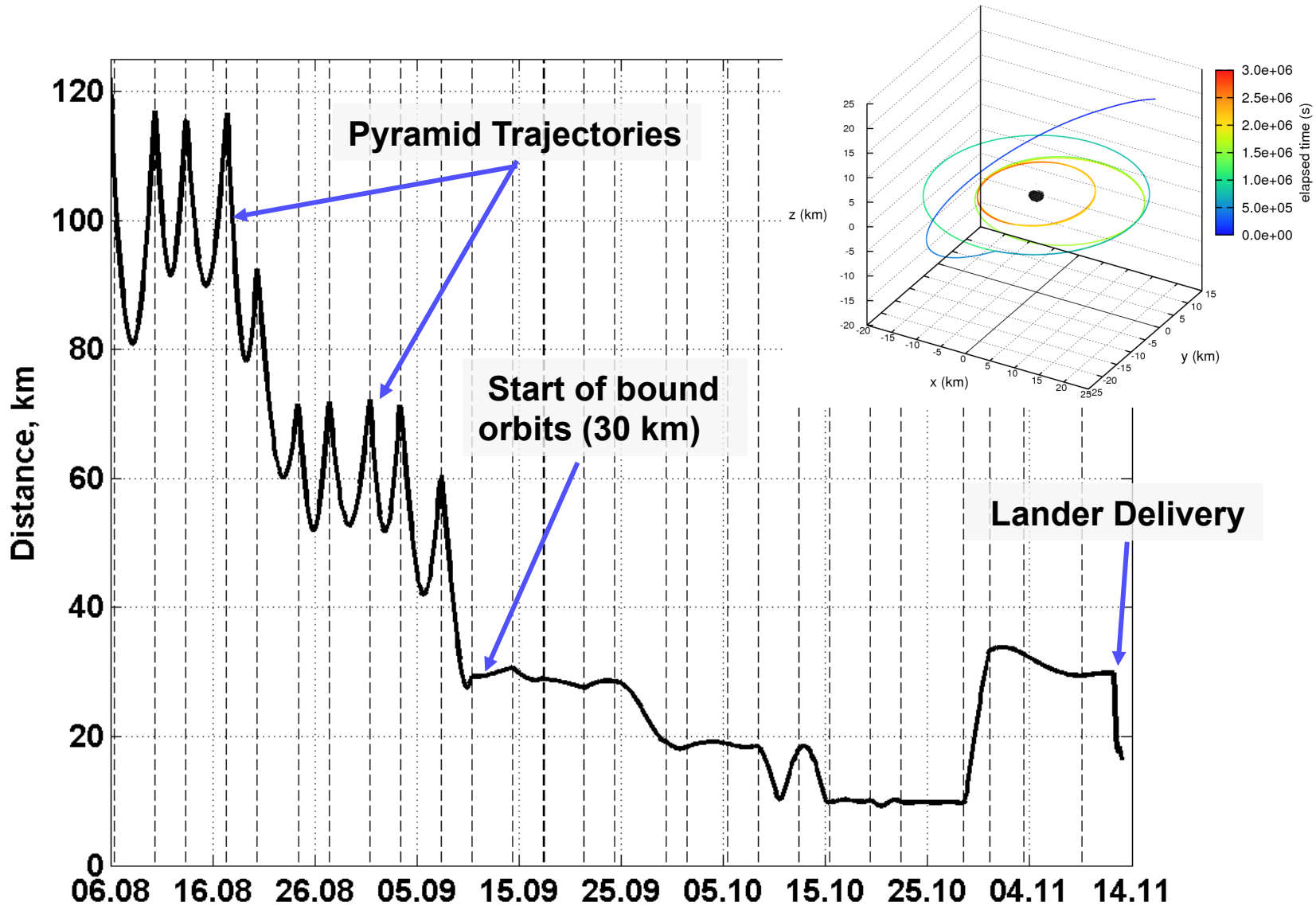
Philae completed its primary science mission after ~ 57 hours on Comet 67P/Churyumov–Gerasimenko





Credit: ESA and OSIRIS Team

Distance from Rosetta to Comet

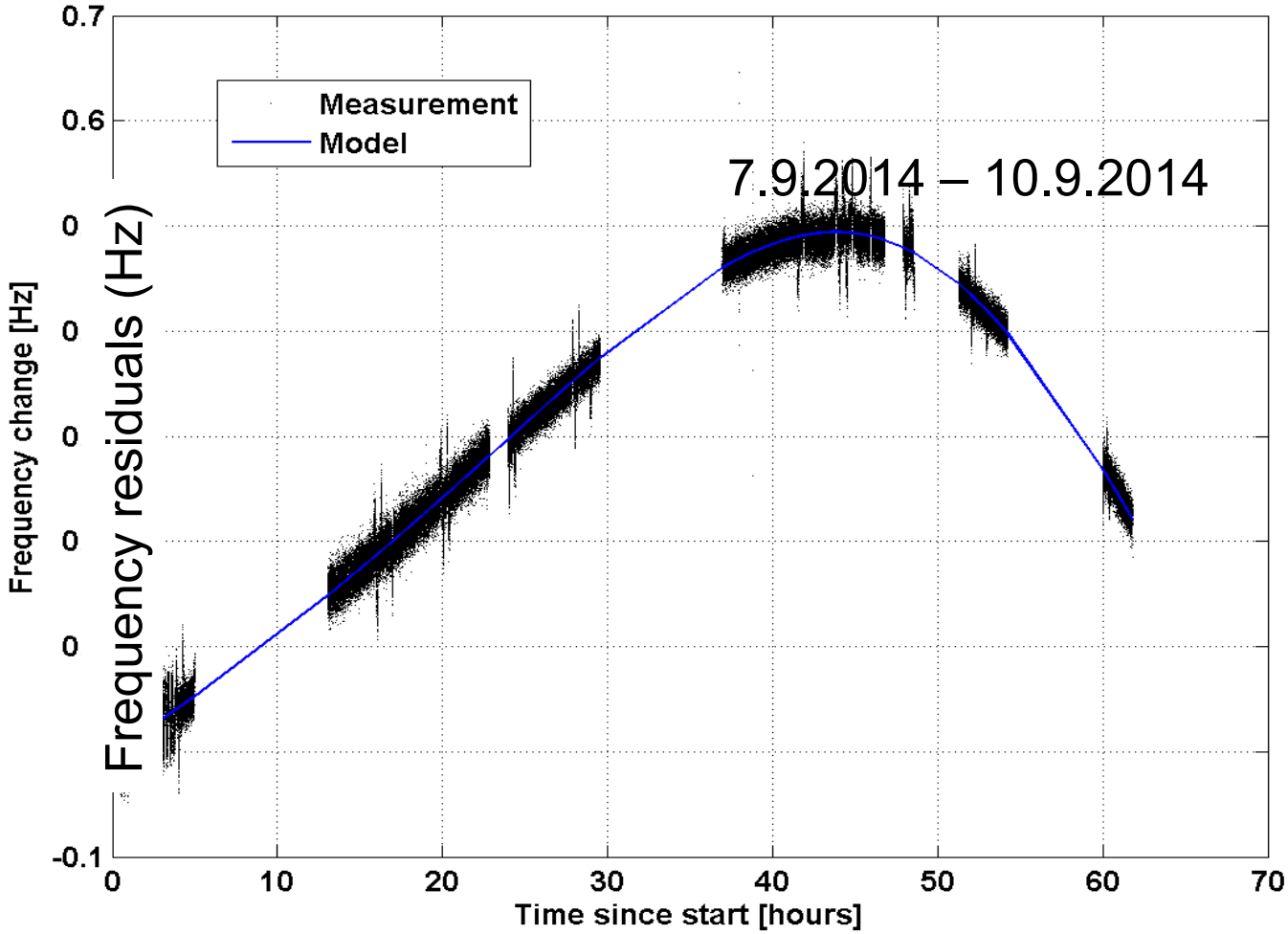


Force Model

- Comet as central body treated (GM and gravity coefficients)
- Comet ephemerides according to latest orbit file from FD
- Sun and planets treated as point masses (*Folkner et al. 2008*)
- J_2 of Sun
- Sun and Planet ephemeris from JPL ephemerides DE423, *Folkner et al. 2008*
- Solar Radiation Pressure (macro model for ROS spacecraft)
- Outgassing is modeled using of Rosina COPS data (density and the total pressure $\Rightarrow$ NG gas velocity)

Nucleus mass determination

September 2014, 30 km bound orbit



Time since 18:28 UT on 7.9.2014 (hours)