Huygens probe entry trajectory reconstruction using Kalman filtering

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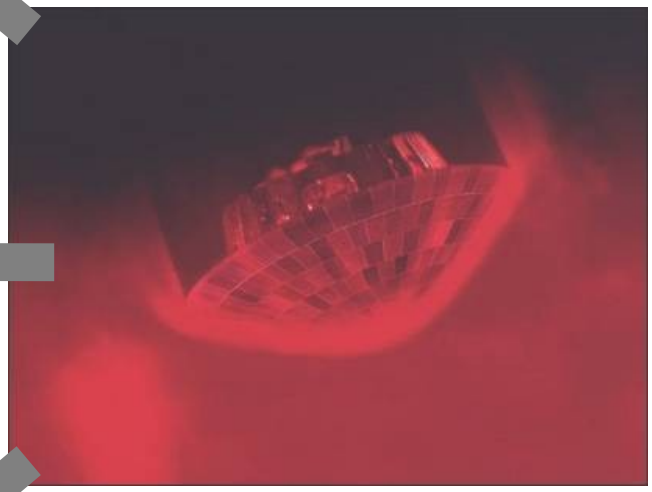
1. Develop a 6DOF state-space model

- Probe CM dynamics (3 DOF)
- Attitude dynamics (3DOF)
- Use of all a-priori information available.

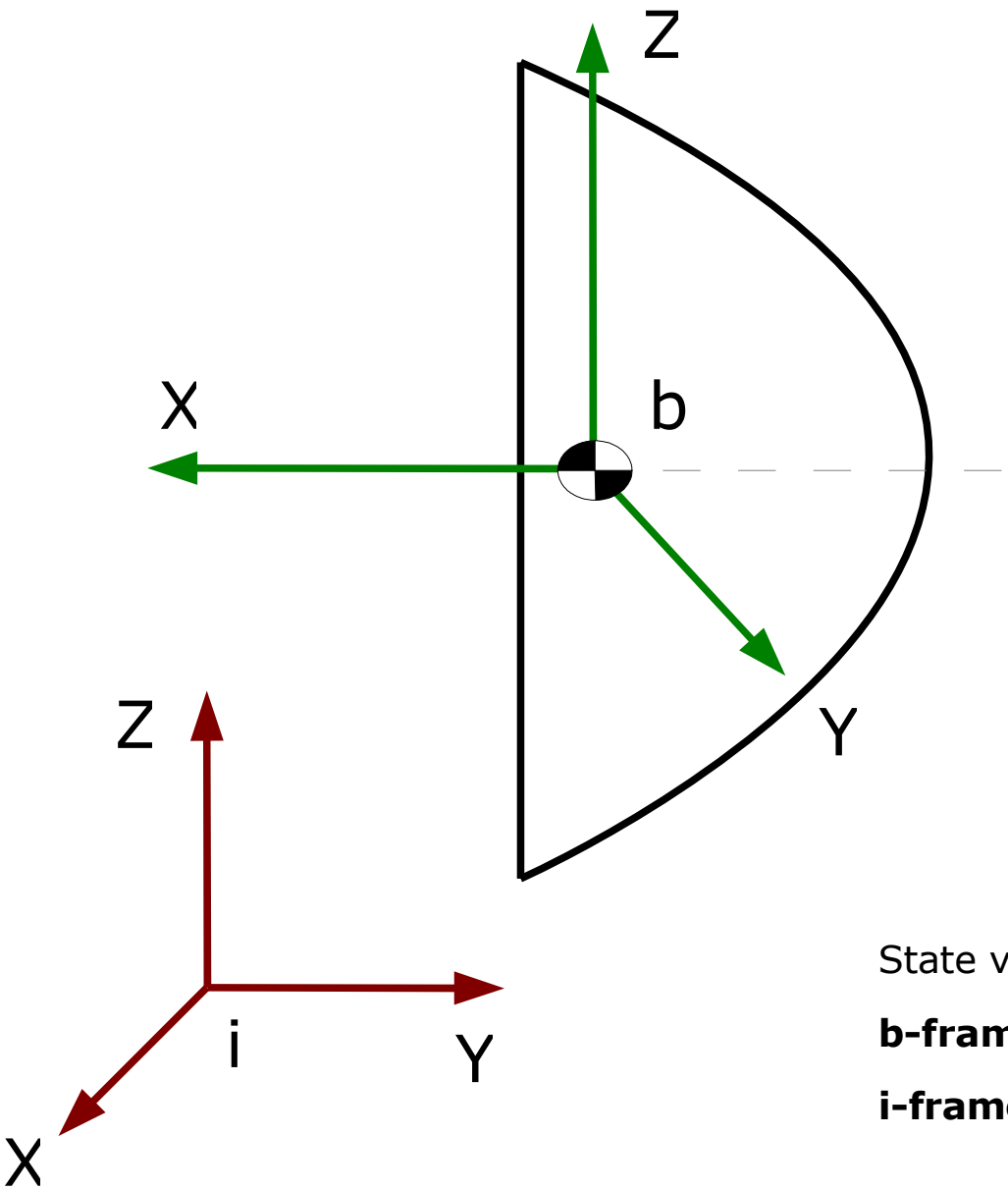


2. Correct interpretation of acquired data

- Initial angular rate estimation.
- Initial altitude and velocity vectors update.

3. Trajectory reconstruction

- Extend the model to include atmospheric effects.
- Reconstruction of position and velocity.
- Estimation of the attitude profile consistent with acceleration data.
- Update of atmospheric density, pressure and temperature profiles.



$$x = \begin{bmatrix} {}^{b}\omega \\ {}^{i}q_b \\ {}^{b}v \\ {}^{i}p \end{bmatrix}$$

${}^{b}\omega$: angular rate

${}^{i}q_b$: attitude quaternion

${}^{b}v$: CM velocity

${}^{i}p$: CM position

State vector elements are expressed wrt

b-frame : probe fixed

i-frame : EME2000, Titan centered, Titan fixed

Forces / troques acting on the probe

Gravitational force

$$ {}^{b}f_{G} = -\frac{GU \cdot M_{T} \cdot M_{P}}{\|{}^{b}p\|^{2}} \cdot {}^{b}e_{P} $$

Dynamic pressure

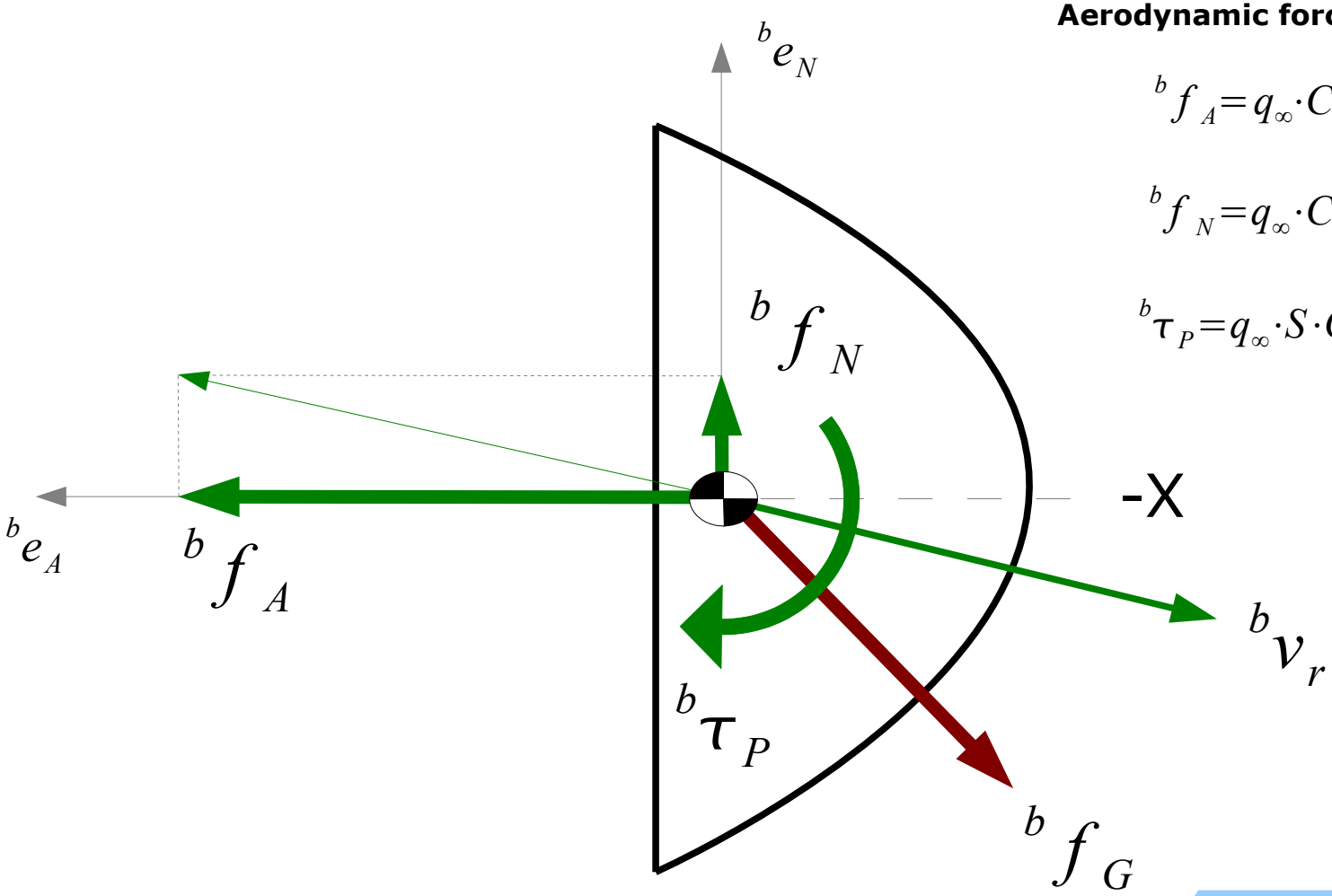
$$ q_{\infty} = \frac{1}{2} \rho v^{2} L $$

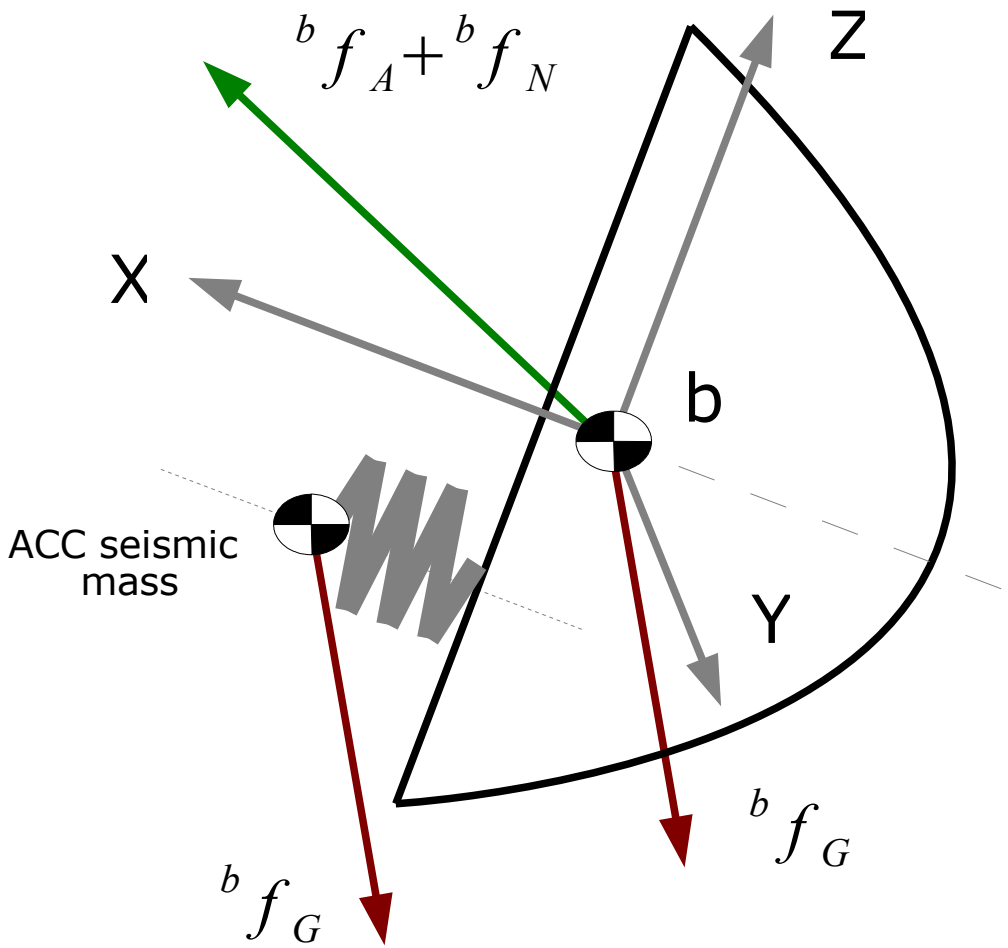
Aerodynamic force/torque

$$ {}^{b}f_{A} = q_{\infty} \cdot C_{A}(\alpha) \cdot {}^{b}e_{A} $$

$$ {}^{b}f_{N} = q_{\infty} \cdot C_{N}(\alpha) \cdot {}^{b}e_{N} $$

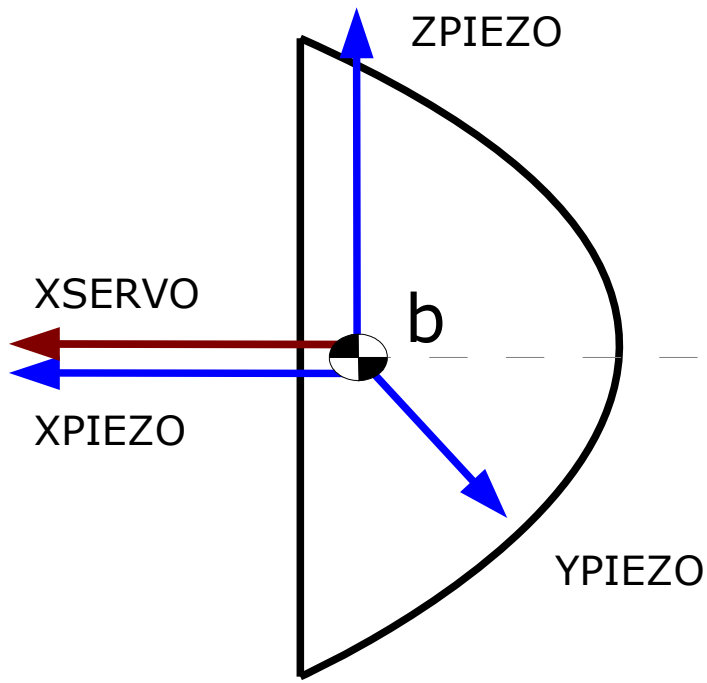
$$ {}^{b}\tau_{P} = q_{\infty} \cdot S \cdot C_{P}(\alpha) \cdot {}^{b}e_{P} $$





ACC measures:
- Aerodynamic forces
- Radial acceleration

$$y = e^T \left\{ \frac{1}{m} \left({}^b f_A + {}^b f_N \right) + {}^b \dot{\omega} \times r + {}^b \omega \times {}^b \omega \times r \right\} + o + \eta$$

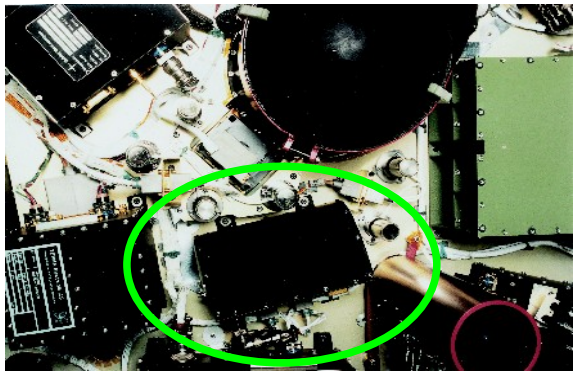


HASI ACC subsystem is composed by four sensors:

1 SERVO accelerometer along b-frame X

3 PIEZO's accelerometers along b-frame X, Y and Z

Only XSERVO sensor has been used for entry reconstruction.



Inputs needed for simulation/reconstruction of entry

Titan

Density, pressure, temperature, mean molecular mass (Post-Ta)

Winds model (Flasar)

Gravitational model (1st order)

Huygens

Heat shield shape

Mass

Inertia tensor

AEDB

Accelerometers

Mounting points

Scale factors, biases

Resolution, uncertainty

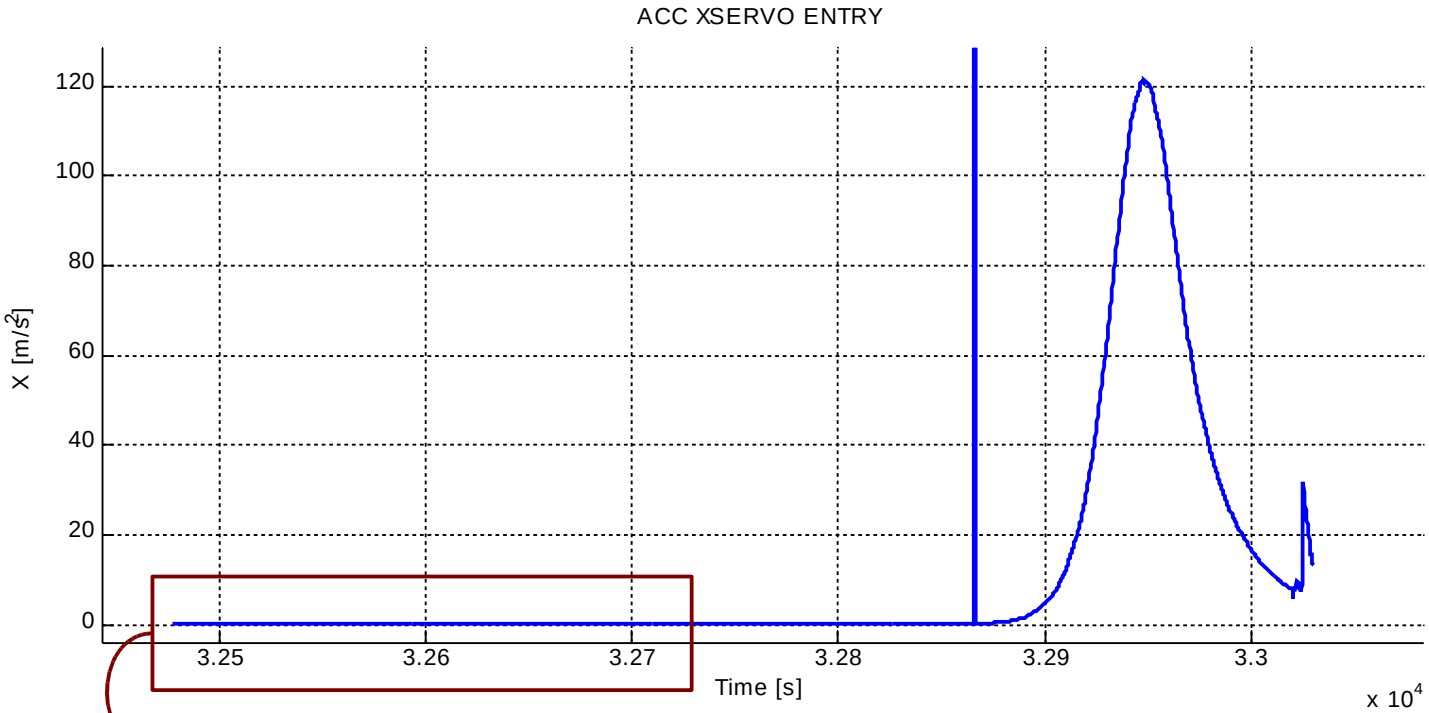
Measurements

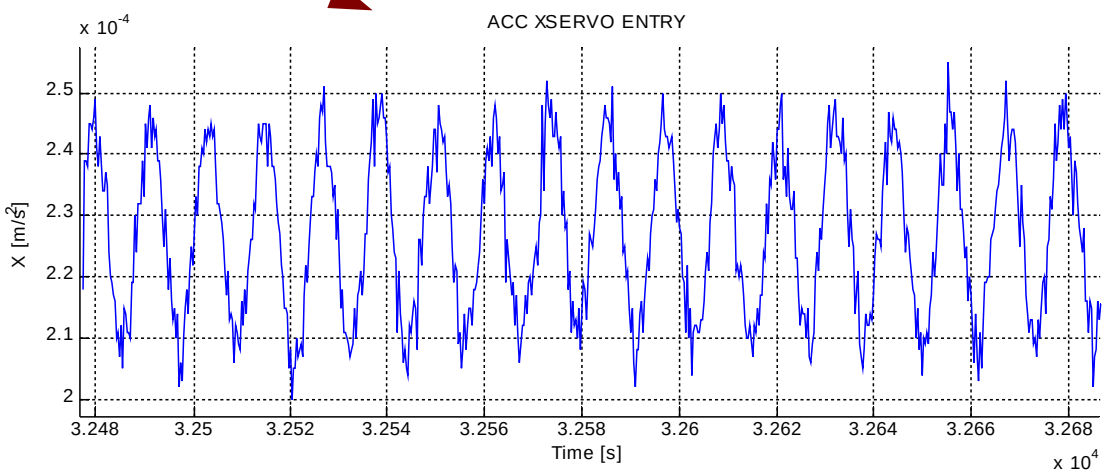
Initial state vector

Position
Velocity → Updated from XSERVO data

Attitude
Angular rate → Estimated from XSERVO data

XSERVO data analysis : acquired data

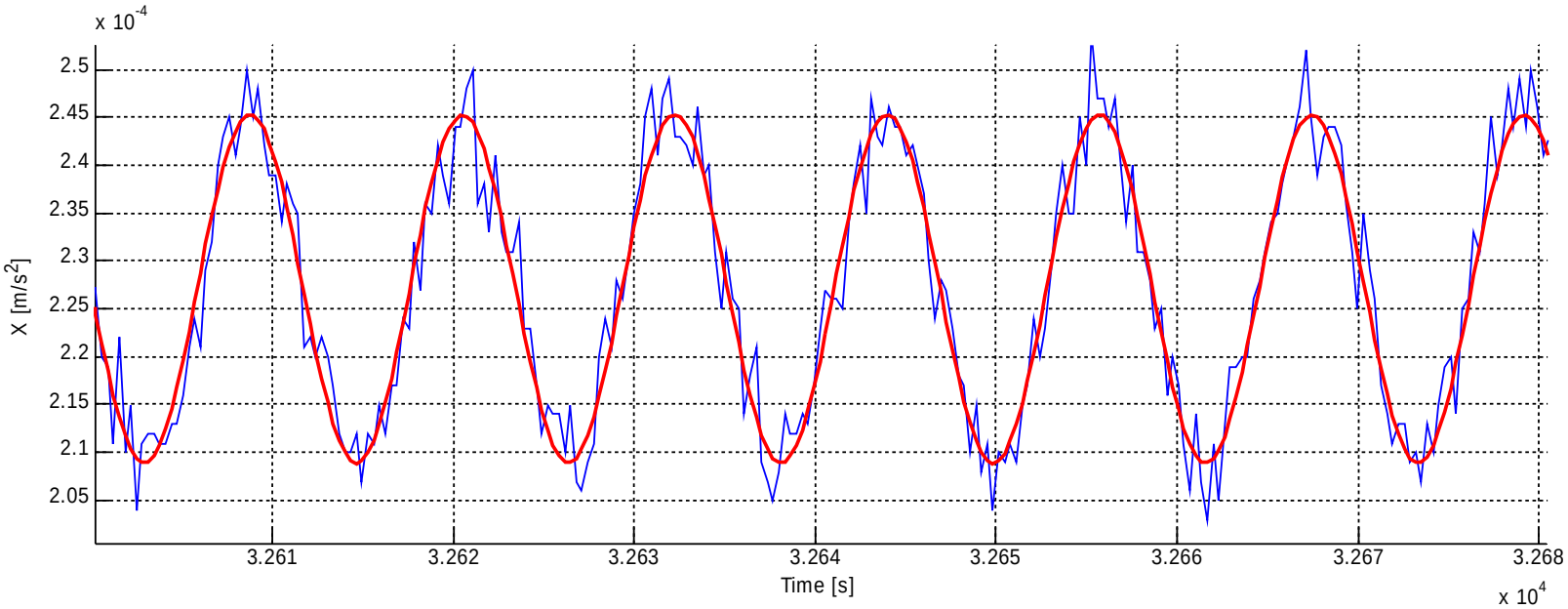




HASI ACC starting from approx. 2800 km

XSERVO measures probe coning motion

XSERVO data analysis : angular rate estimation



$$ {}^{b}\dot{\omega} = {}^{b}I\left\{0 - {}^{b}\omega \times {}^{b}I\,{}^{b}\omega\right\} $$

$$ y = s^{T}\left\{0 + {}^{b}\dot{\omega} \times r + {}^{b}\omega \times {}^{b}\omega \times r\right\} + o $$

	Angular rate rad/s	
	@ 1570 km	@ 1270 km
X	0.73169	0.73174
Y	0.00848	0.00495
Z	-0.00696	0.00476

XSERVO data before entry reflects probe coning motion.

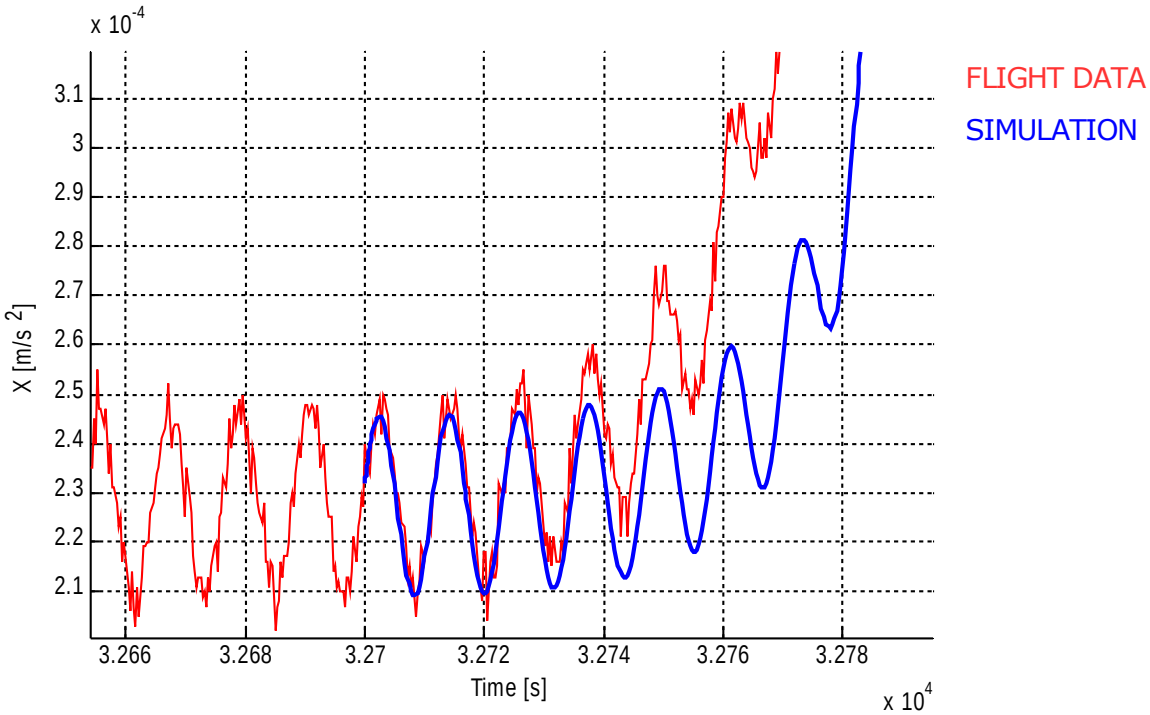
(it seems a sinusoid but it is not and depends ONLY from initial angular rate)

A torque free spinning body model can be used to fit XSERVO data.

(using a Non Linear Least Square technique)

6.99 RPM around X axis

XSERVO data analysis : angular rate estimation



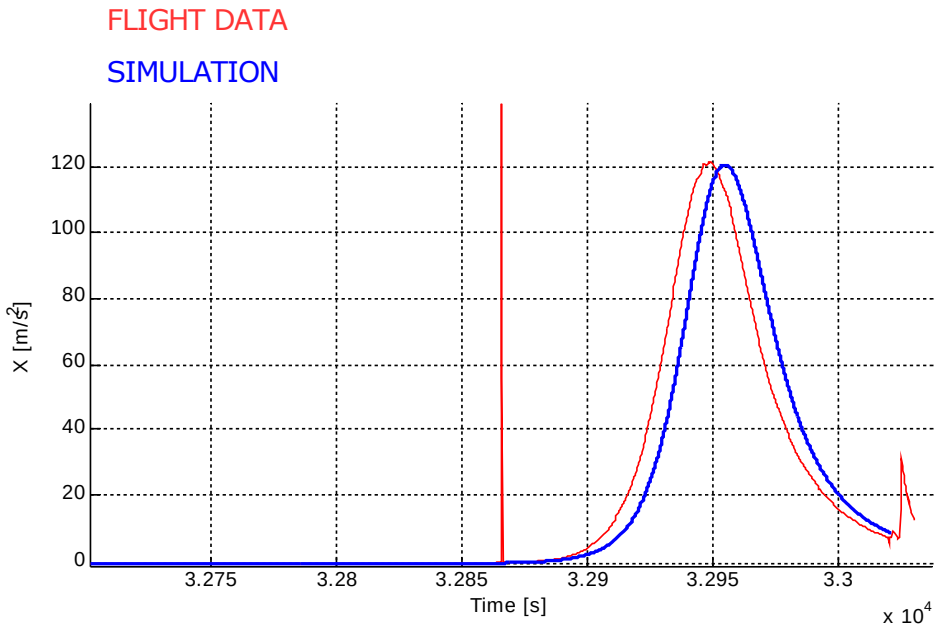
Probe coning is related only to

- initial angular rate
- probe inertia tensor

using estimated angular rate simulated XSERVO data fit very well flight data

... for coning component ...

XSERVO data analysis : update initial conditions



XSERVO 1570 km simulated data match well measurements:

- Magnitude
- Shape

About 6.7 s time lag is evident:

- Initial altitude is probably lower than the one from entry state vector.

Entry EME2000 position as given by Cassini Navigation Team can be updated to match XSERVO measurements:

Uncertainty in a-priori density profile

approx. 5 km

Entry state vector derived altitude uncertainty

approx. 32 km

Modelled density profile is given with 10 km resolution.

XSERVO data analysis : update initial conditions

EME2000 [m]

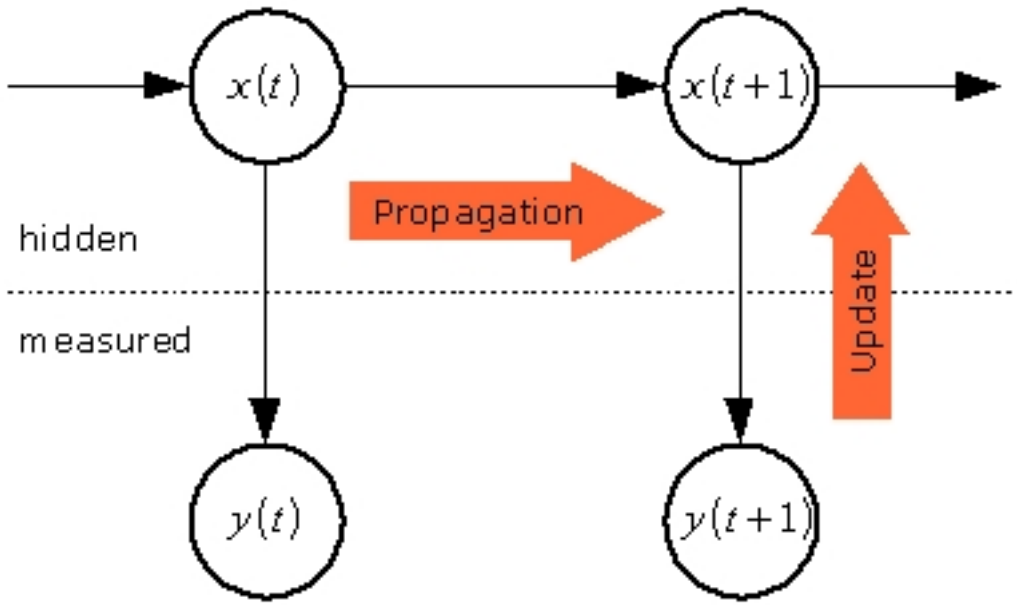
X	44712	→	44364
Y	-4132632		-4100480
Z	-320197		-317706

Altitude [km]

1570.259	→	1538.009	(-32.250)	HASI
1270.011		1248.423	(-21.588)	Interface altitude

Correction was made manually keeping in mind 1-sigma uncertainty interval!

Profiles seem to be poorly dependent on initial velocity (1-sigma interval).



State vector:

- Attitude
- Position
- Velocity

Measurement:

- Acceleration

Kalman filter estimates the hidden states of a system processing all available measurements regardless of their precision and quantity with use of:

- knowledge of dynamics of the system and measurement devices.
- statistical description of the system uncertainty and measurement errors.
- any available information about the initial conditions.

Titan's atmospheric density profile can be different from the tabulated one!

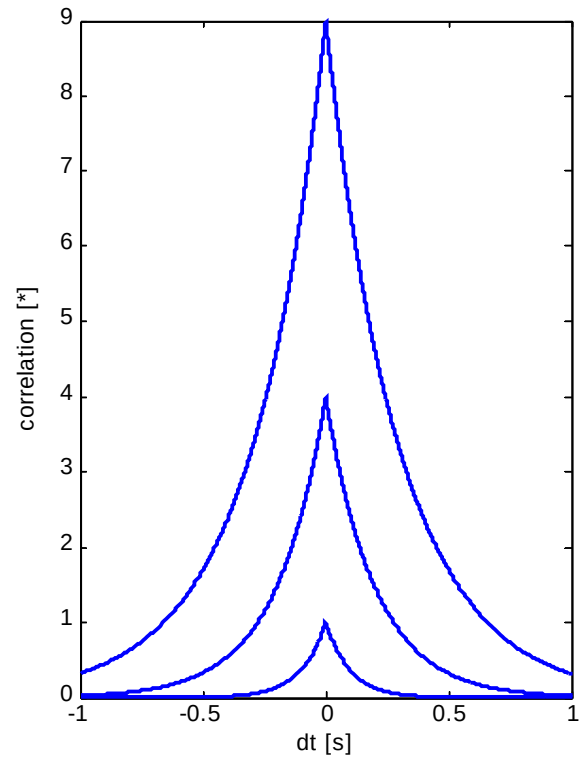
$$\rho_r = \rho_t + \delta_\rho$$

Real Tabulated Drift

There's no way to define a deterministic model of the drift.

The simpler solution is to represent it using a first order Gauss-Markov stochastic process

$$\dot{\delta}_\rho = -\frac{1}{\beta}\delta_\rho + \epsilon_\delta$$



Aerodynamic properties of the probe vary with density, pressure and temperature, so it must be integrated in the state model.

Kalman filtering solution: extended state space model

State equations

Simulation

$$ {}^b\dot{\omega} = {}^bI\left\{{}^b\tau_P - {}^b\omega \times {}^bI\,{}^b\omega\right\} + \epsilon_\omega $$

$$ {}^b\dot{q}_i = \frac{1}{2}\Omega\left({}^b\omega\right){}^bq_i $$

Attitude

$$ {}^b\dot{v} = \frac{1}{m}\left({}^bf_A + {}^bf_N + {}^bf_G\right) - {}^b\omega \times {}^bv + \epsilon_v $$

$$ {}^i\dot{p} = {}^iR_b\,{}^bv $$

Position

$$ \dot{\delta}_\rho = -\frac{1}{\beta}\delta_\rho + \epsilon_\delta $$

$$ \dot{P} = \rho\,{}^bf_G^T \cdot {}^bv $$

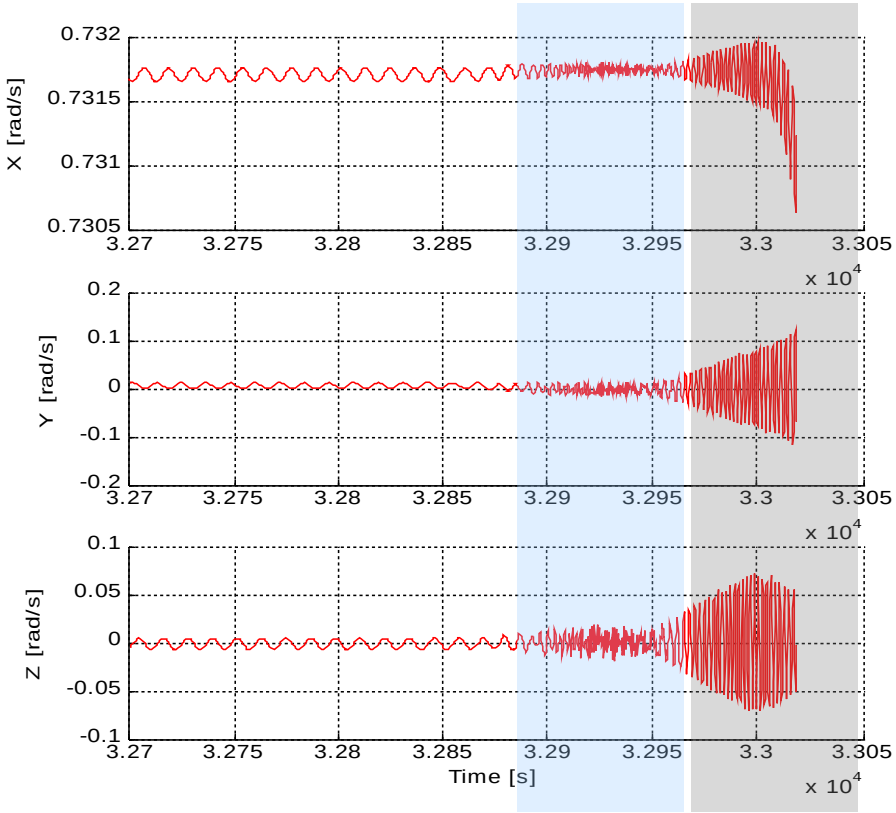
Atmosphere

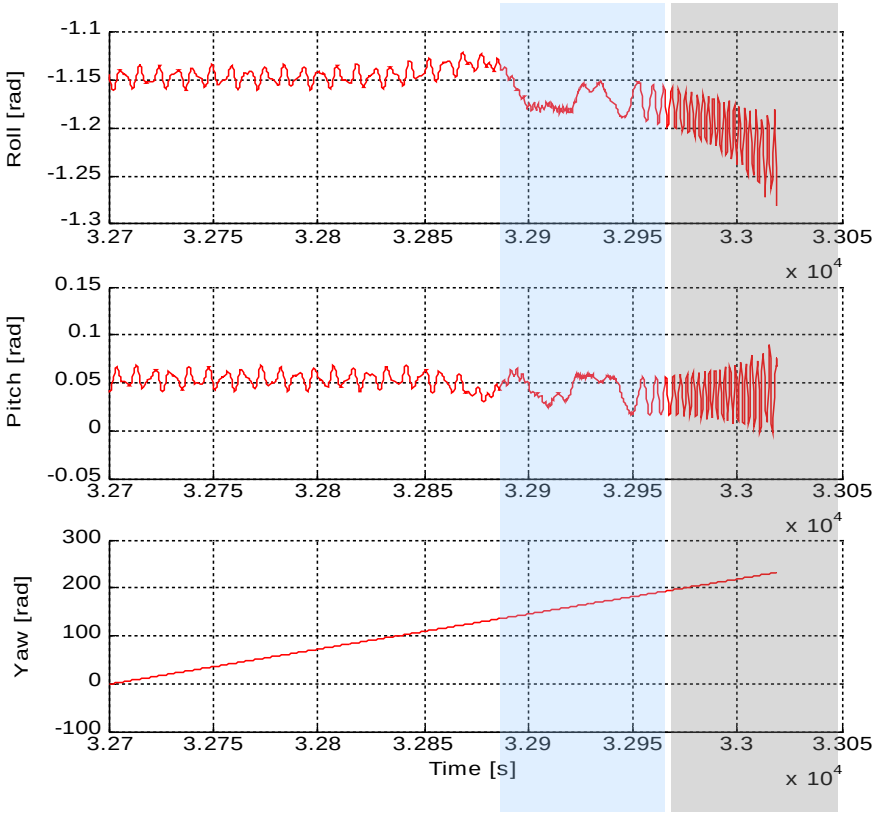
Reconstruction

Measurement equation

$$ y = e^T\left\{\frac{1}{m}\left({}^bf_A + {}^bf_N\right) + {}^b\dot{\omega} \times r + {}^b\omega \times {}^b\omega \times r\right\} + o + \eta $$

Trajectory reconstruction: angular rate and attitude





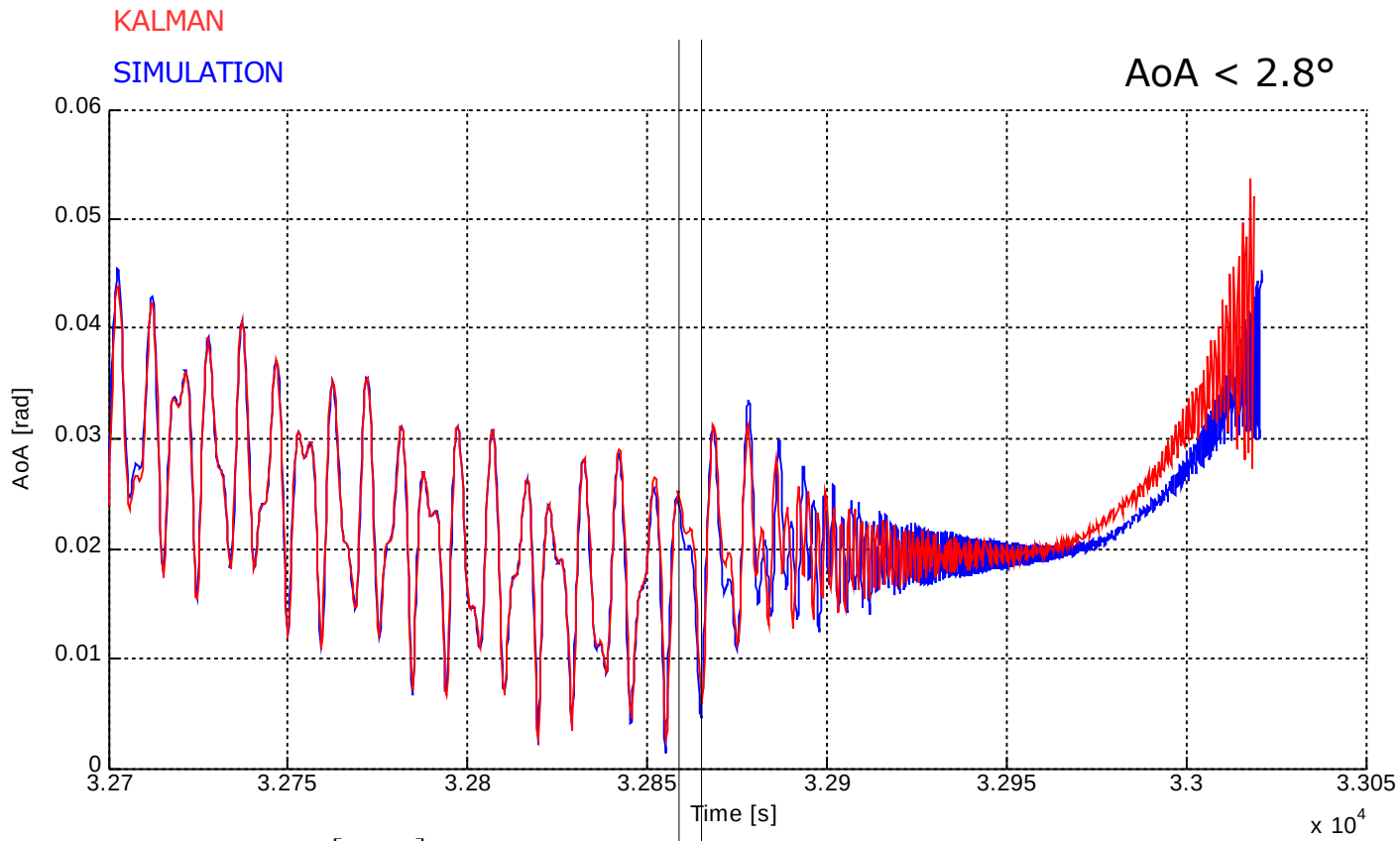
$\sigma_{max} = 0.014 [rad/s]$

$\sigma_{max} = 0.026 [rad]$

Effects of pitching moment and the terminal dynamical instability are evident

XSERVO measurements are not sufficient to reconstruct position and attitude of a vehicle (at least 6 acc are required), but the 6DOF model of the probe can be used to estimate an attitude profile **consistent** with measurements.

Trajectory reconstruction: angle of attack

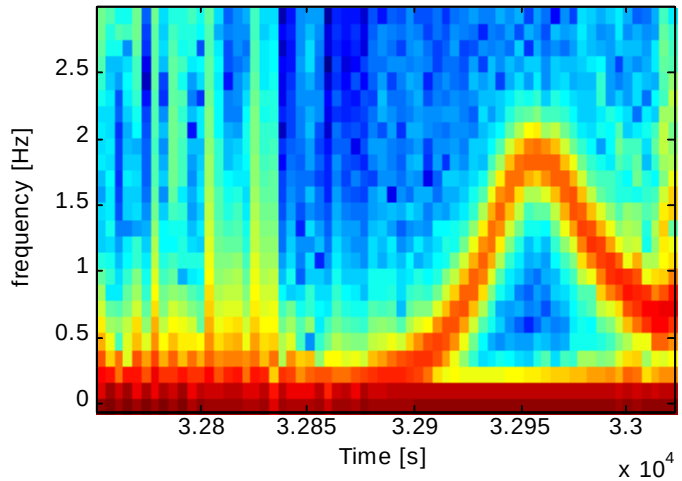


$$\sigma_{max} = 0.028[rad]$$

Using only XSERVO measurements, before entry the algorithm is ruled by the dynamical model of the probe

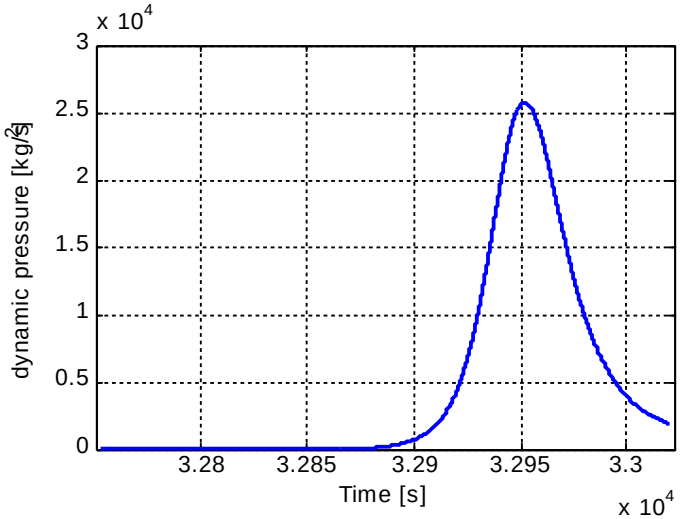
During entry, thanks to aerodynamic forces, the attitude can be updated in a consistent way with respect to measurements.

Trajectory reconstruction: angle of attack



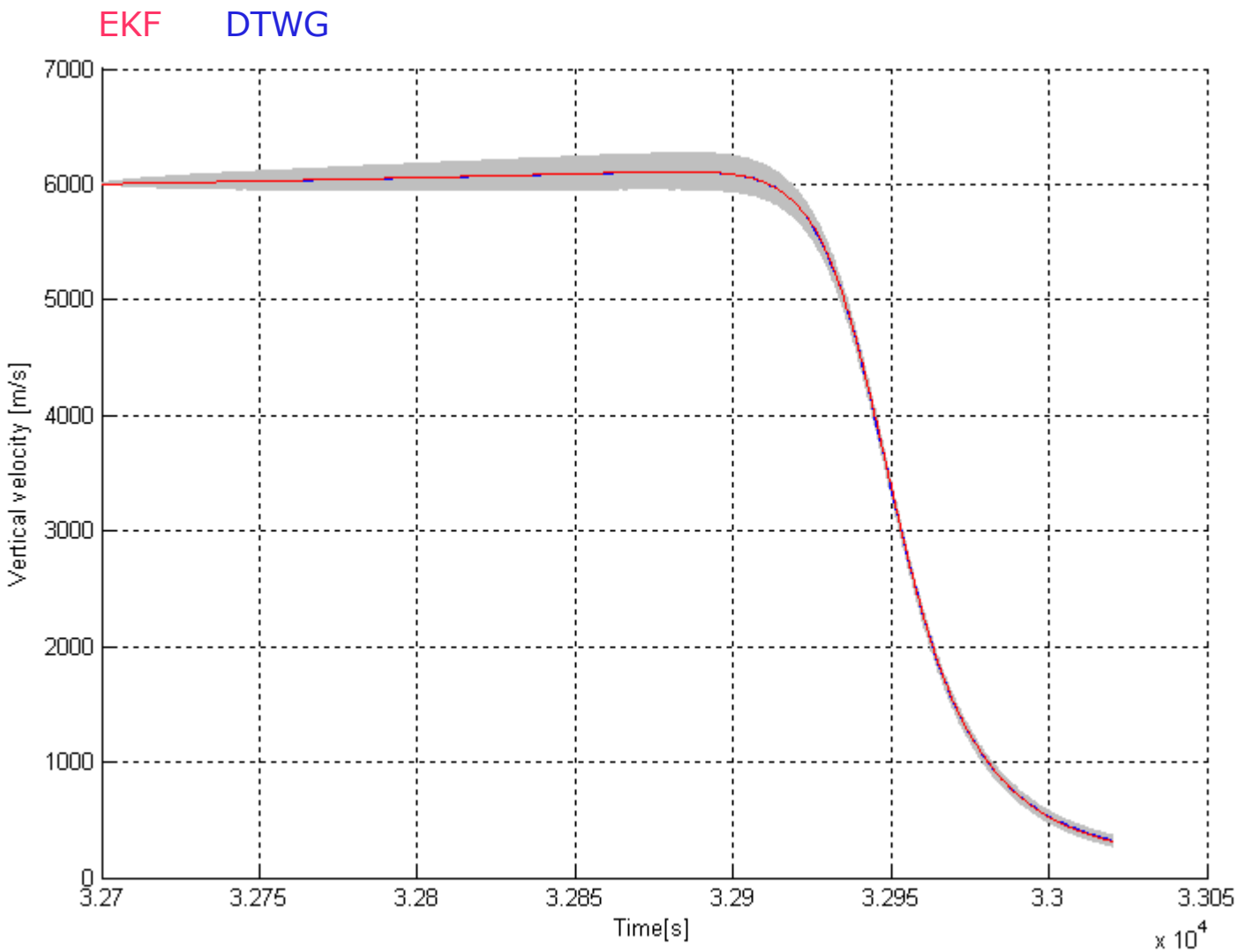
Simulated/reconstructed AoA shows evidence of terminal instability phenomena

AoA frequency is related with dynamic pressure as stated by theory

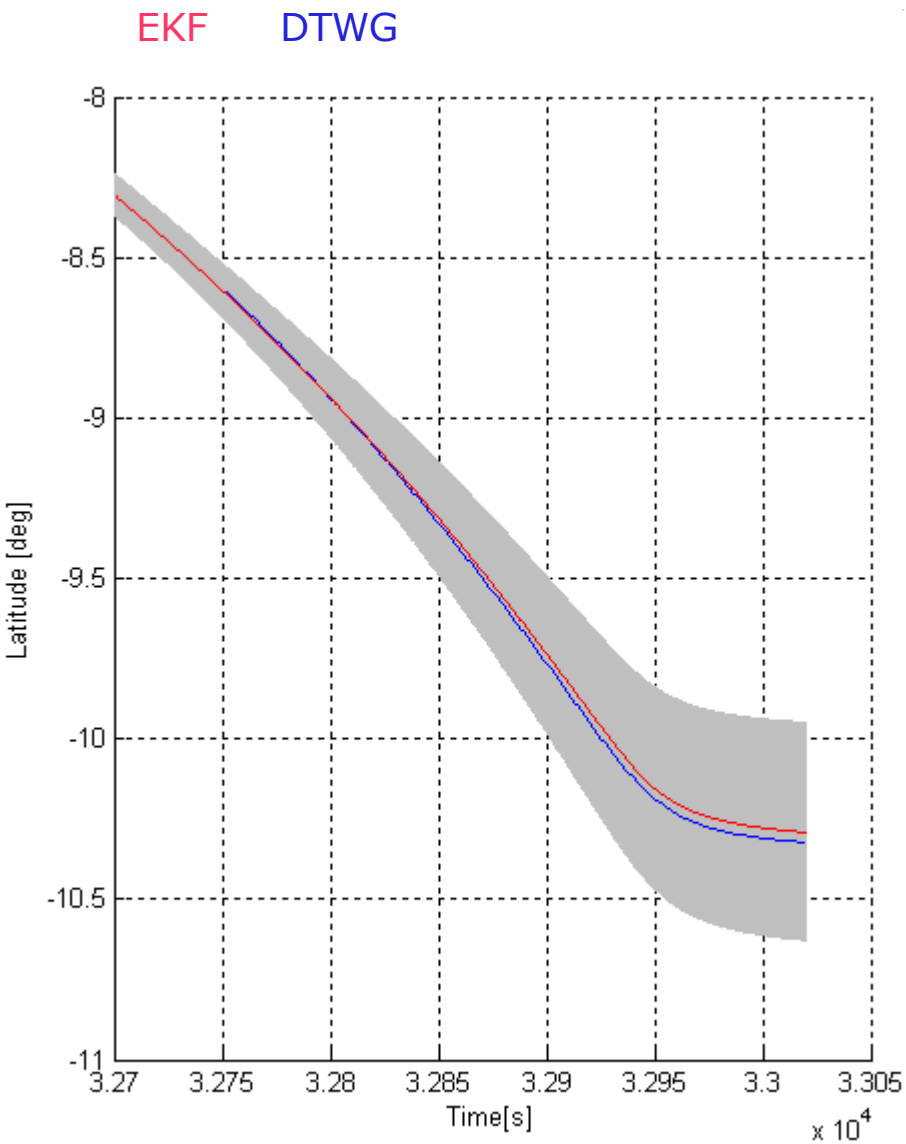


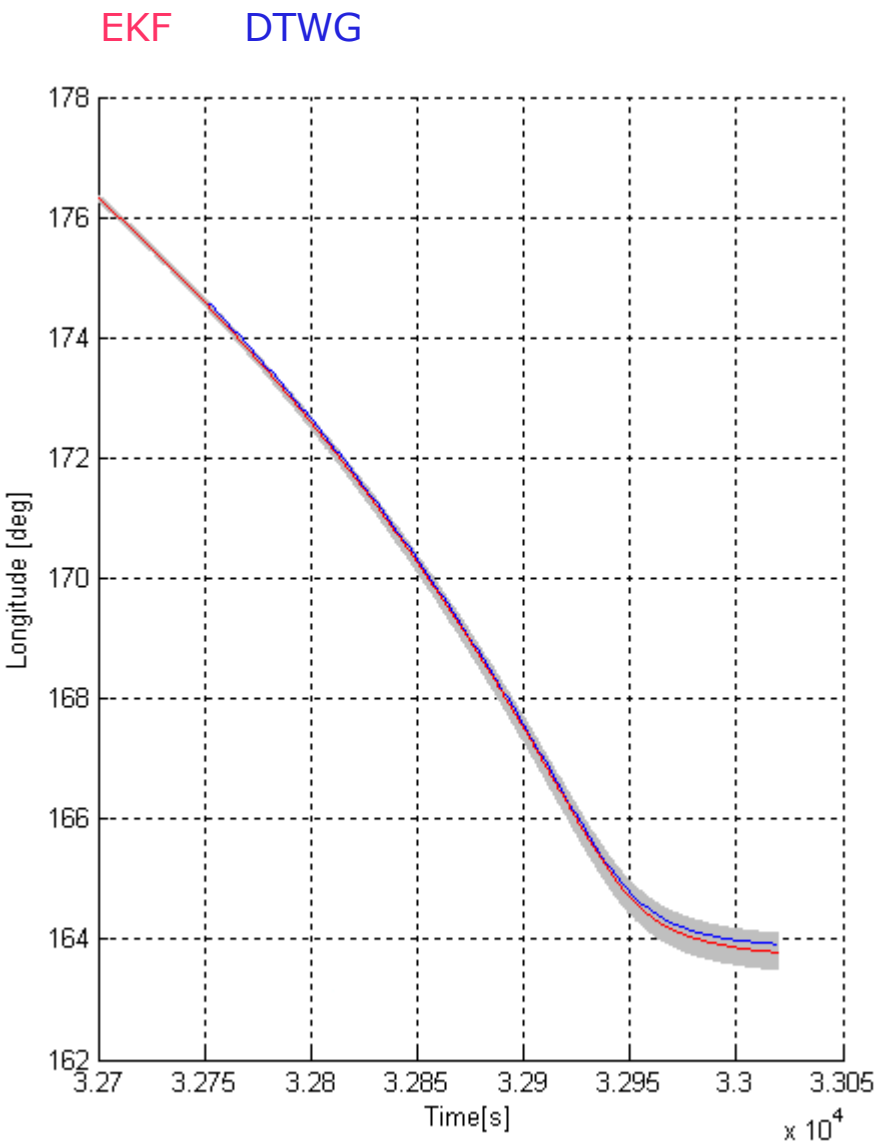
It confirms that aerodynamic effects (force / torque) are well modeled

Trajectory reconstruction: vertical velocity

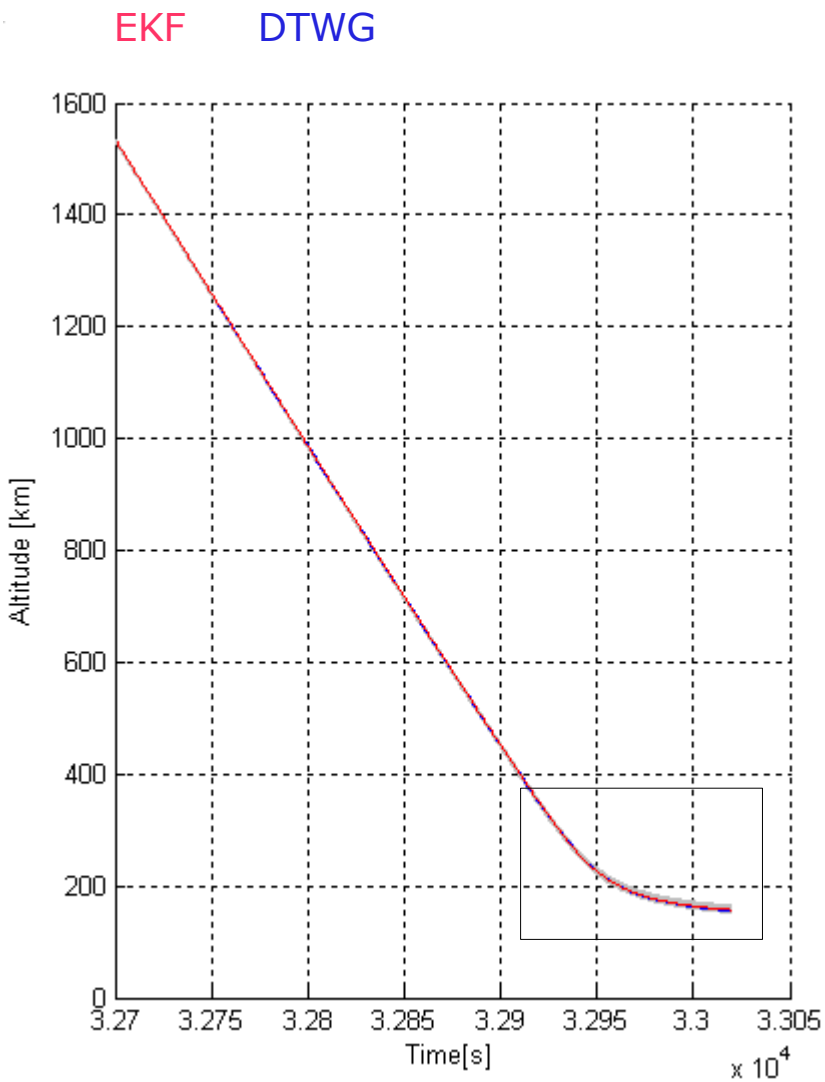


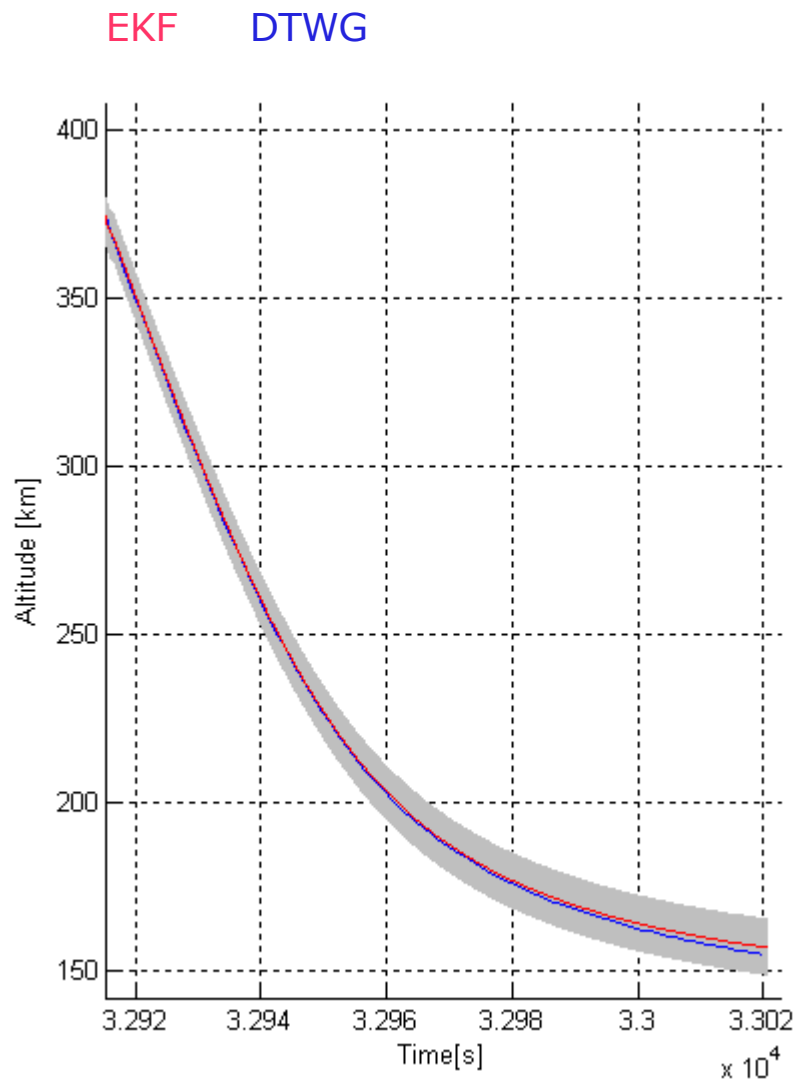
Trajectory reconstruction: latitude and longitude





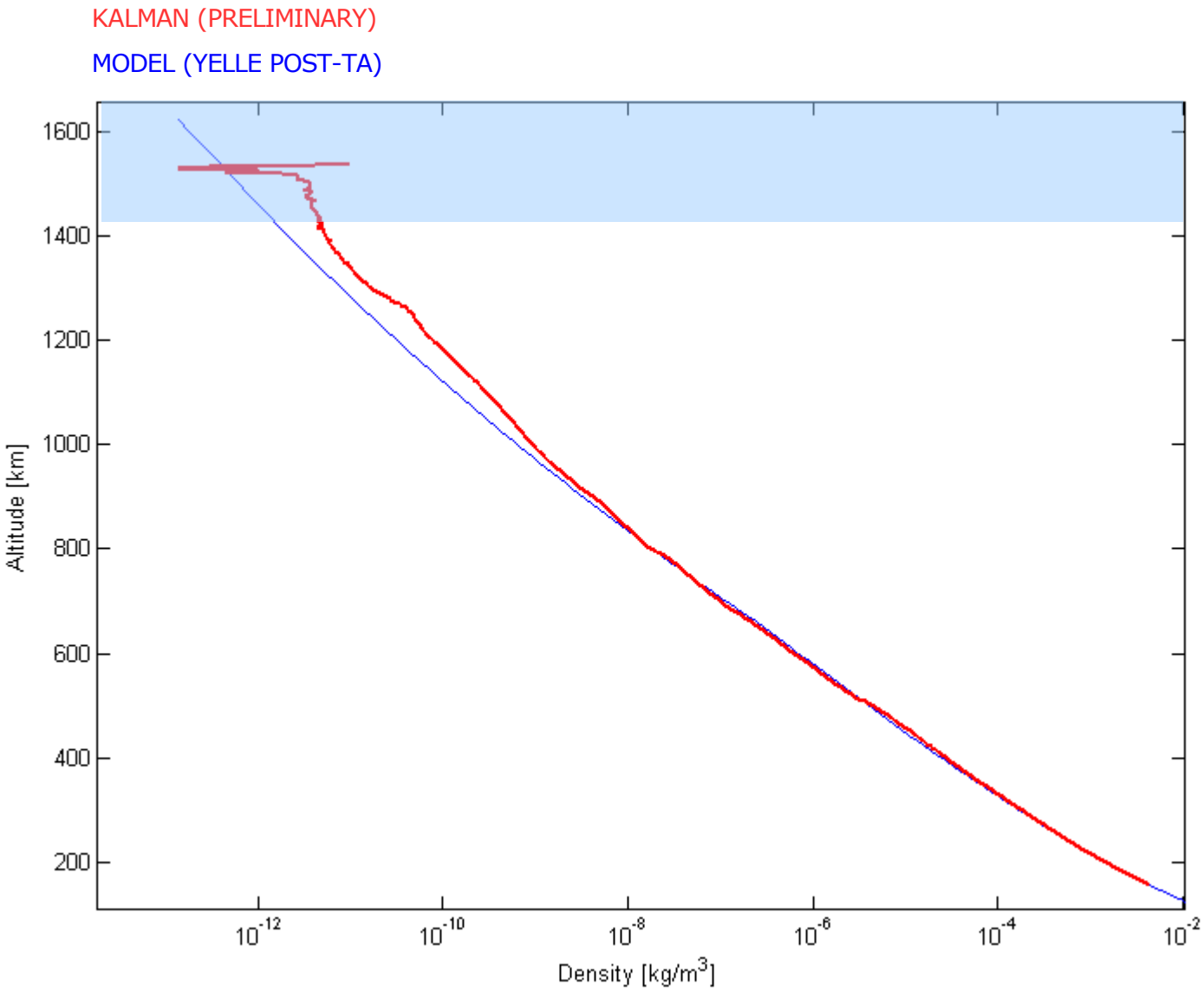
Trajectory reconstruction: altitude



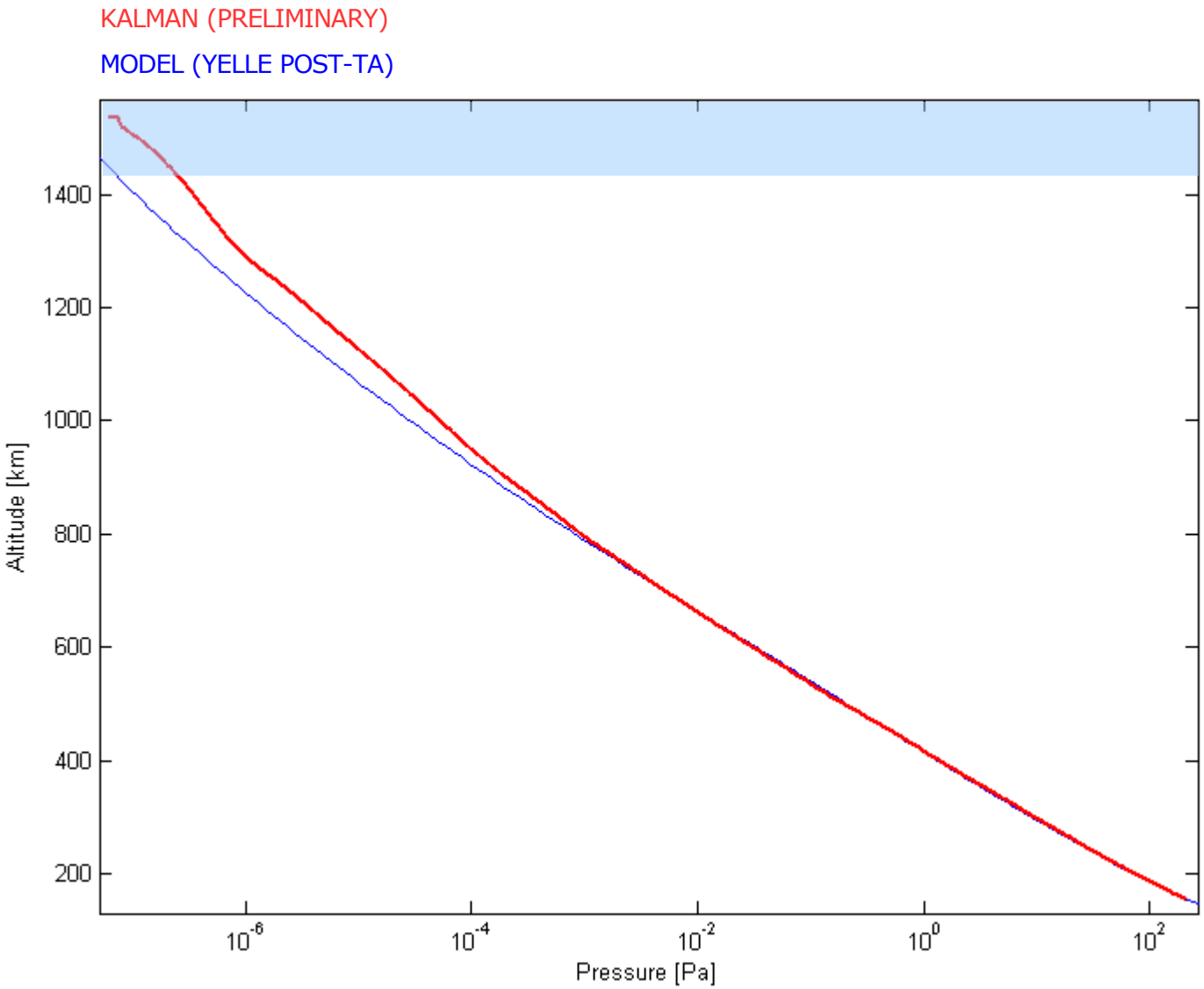


Final altitude is 155.92 [km] with 6.34 [km] 1-sigma unc.

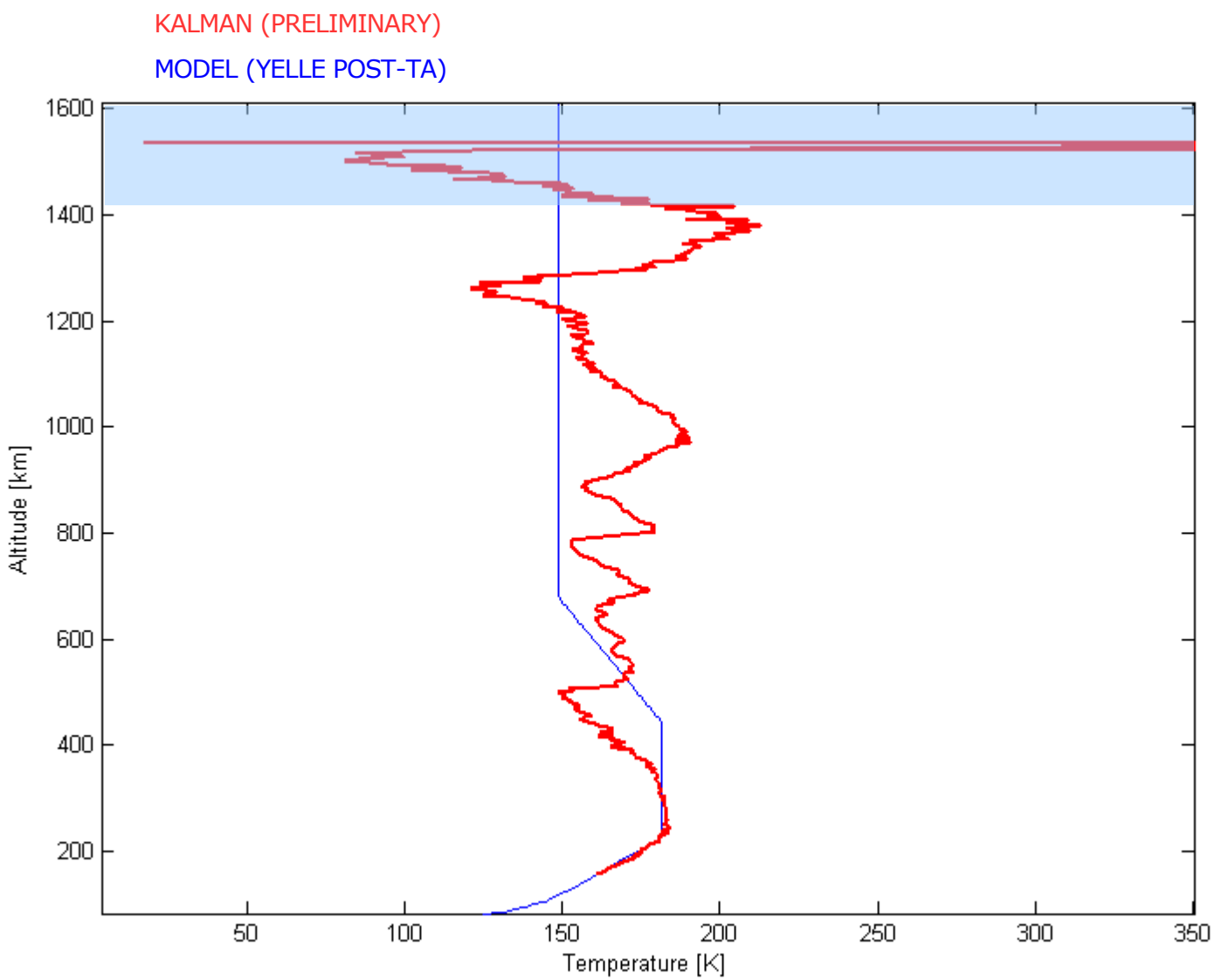
Trajectory reconstruction: density profile



Trajectory reconstruction: pressure profile



Trajectory reconstruction: temperature profile



$$p \qquad \rho_t$$

$$\downarrow$$

$$T = \frac{p \cdot \mu}{\rho_t \cdot R}$$

A **very accurate ACC** and a **phisical model** of the probe can be used to:

- Update initial conditions.
- Reconstruct probe trajectory.
- Compute an attitude profile consistent with measurements.
- Reconstruct density, pressure and temperature profiles of Titan's atmosphere where **no direct measurements are possible**.

Kalman filter requires a very carefull tuning.

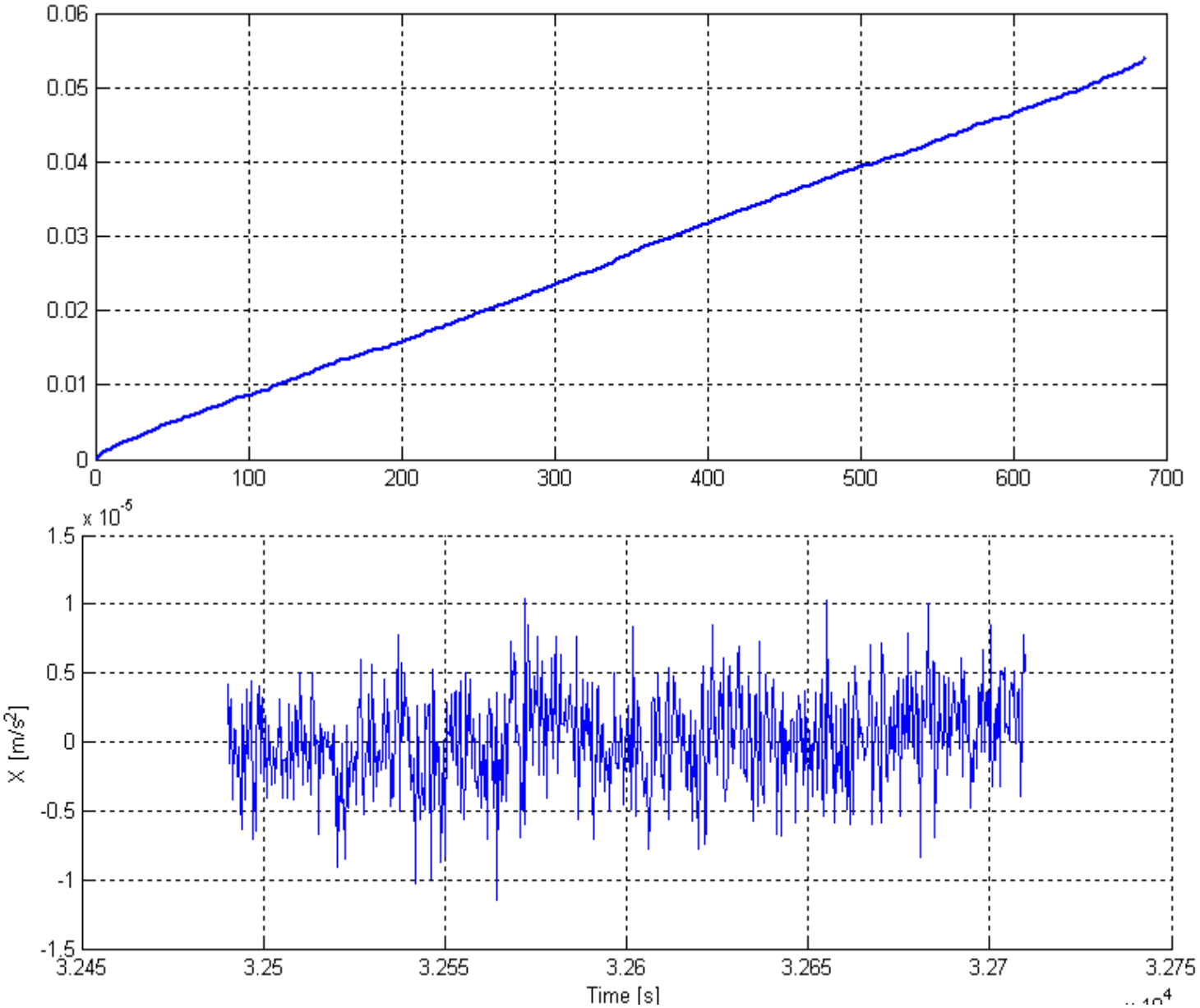
- Challenging!

Algorithm validation can be done only by simulations.

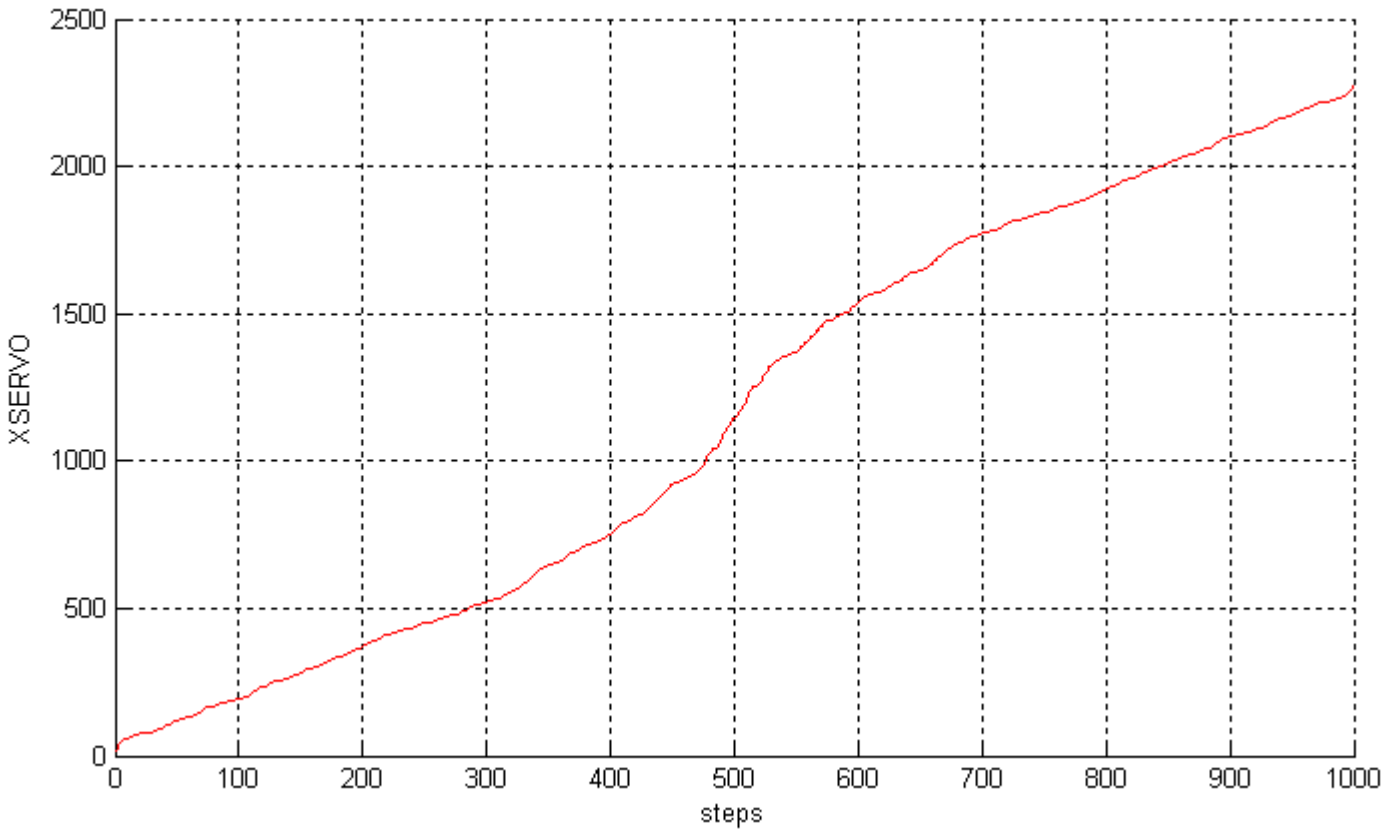
- Experimental activities are difficult to do for entry.

- **Initial conditions** are key parameters for reconstruction.
- **AEDB resolution and accuracy** must be optimized for expected dynamical range!
- When selecting a reconstruction algorithm/approach:
  - Identify the "tuning" parameters and how they can be estimated (if possible).
  - Define validation procedure (simulation and experimental).
- Multiple reconstruction algorithms approach allowing **comparison and verification of results.**

ANNEX 1 : coning residuals whiteness test



ANNEX 2 : XSERVO residuals whiteness test



Max residual @ peak : 0.6 m/s^2

Main objective:

to measure the **Huygens probe's acceleration**

and thus to derive **Titan's atmospheric density profile**.

	Range		Uncertainty	
	High gain	Low gain	High gain	Low gain
High res	+- 0.018	+- 0.180	18e-5	1.8e-3
Low res	+- 18	+- 180	0.18	1.8

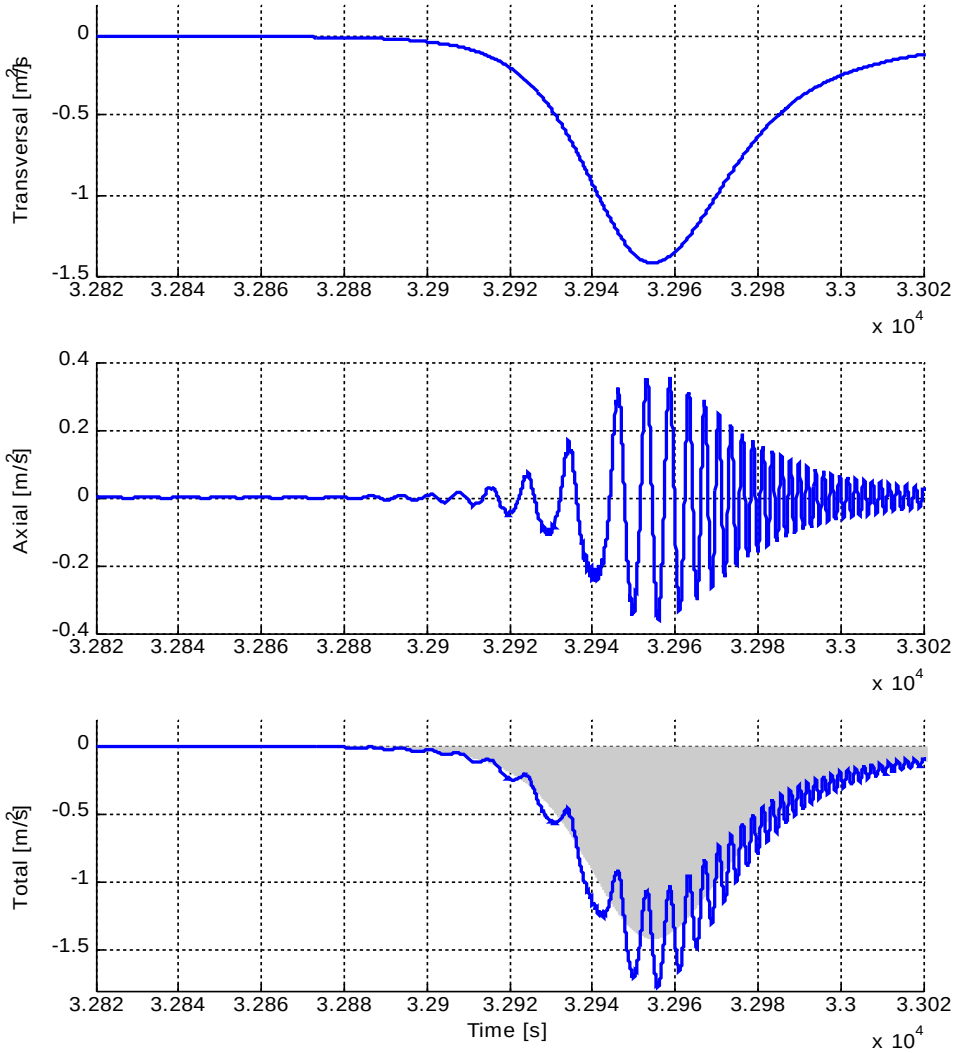
all values are in m/s^2

Resolution: 9.8e-6 to 9.8e-5 m/s^2 (mode dependent)

High resolution allows to measure probe coning motion

Sampling rate: 3.125 Hz

ANNEX 4 : PIEZO data analysis - expected measurements



Accelerometer measurements are sum of three components:

Transversal sensitivity

(reflects deceleration profile)

+

Radial acceleration

(approx. 10e-4 negligible)

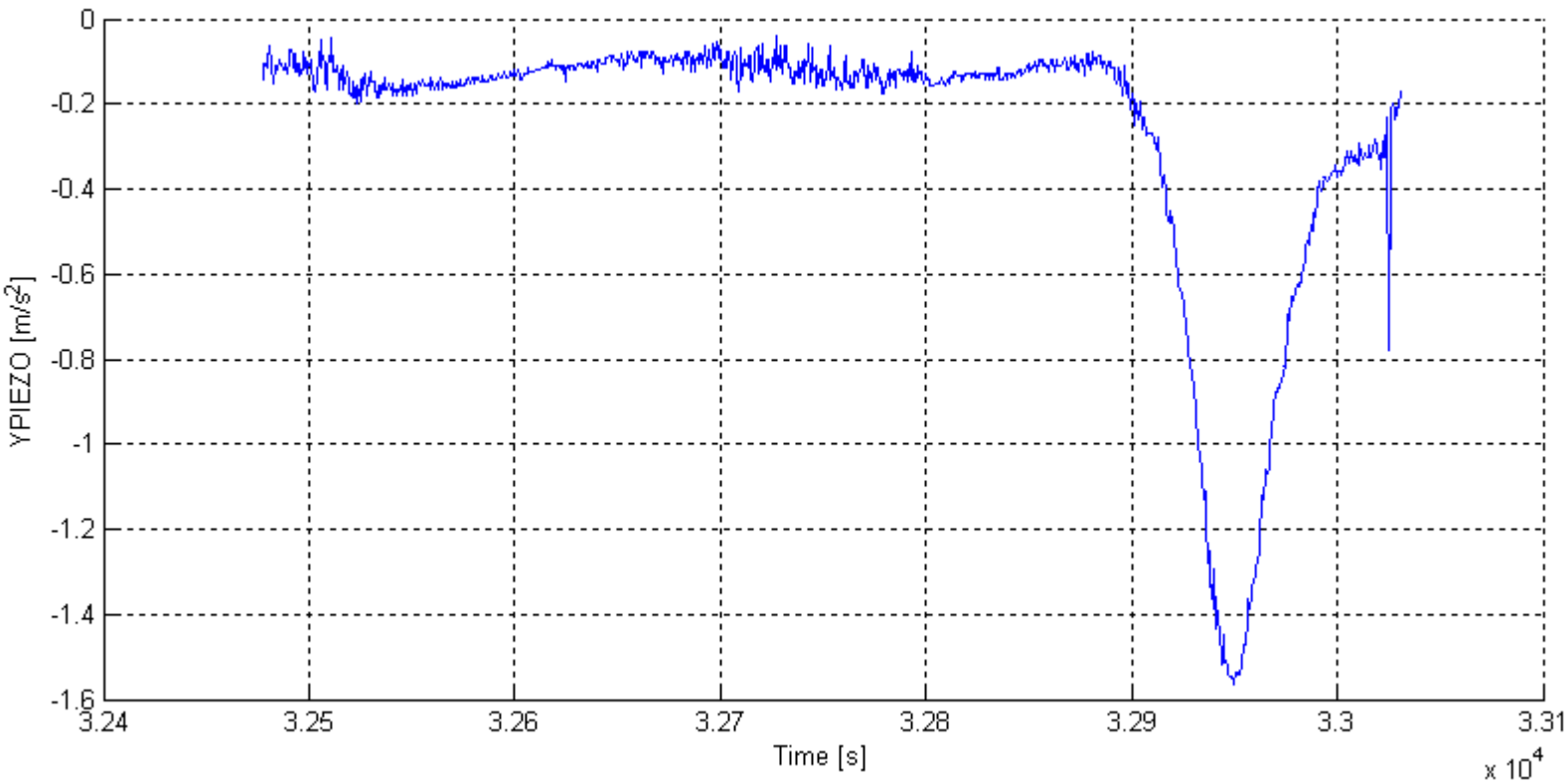
+

External forces (aerodynamic)

(modulated by probe spin)

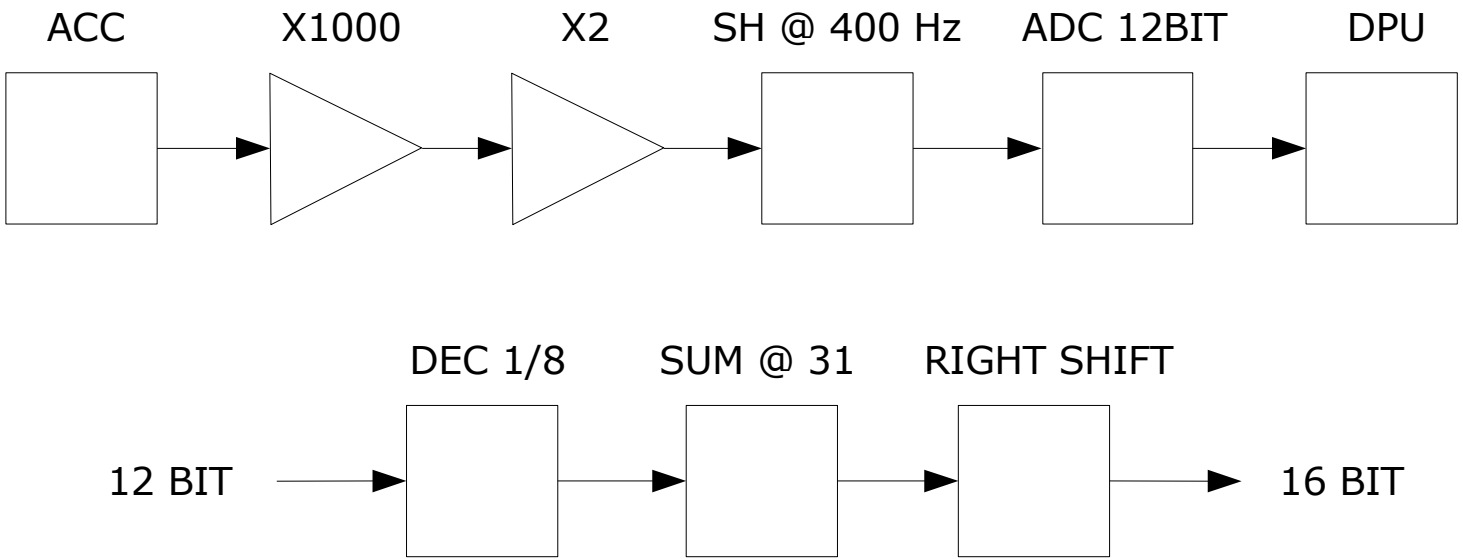
=

Total measured acceleration



Where's normal forces !?

ANNEX 4 : PIEZO data analysis - measurements chain



SENSIBILITY : 0.0408163 V/m/s^2

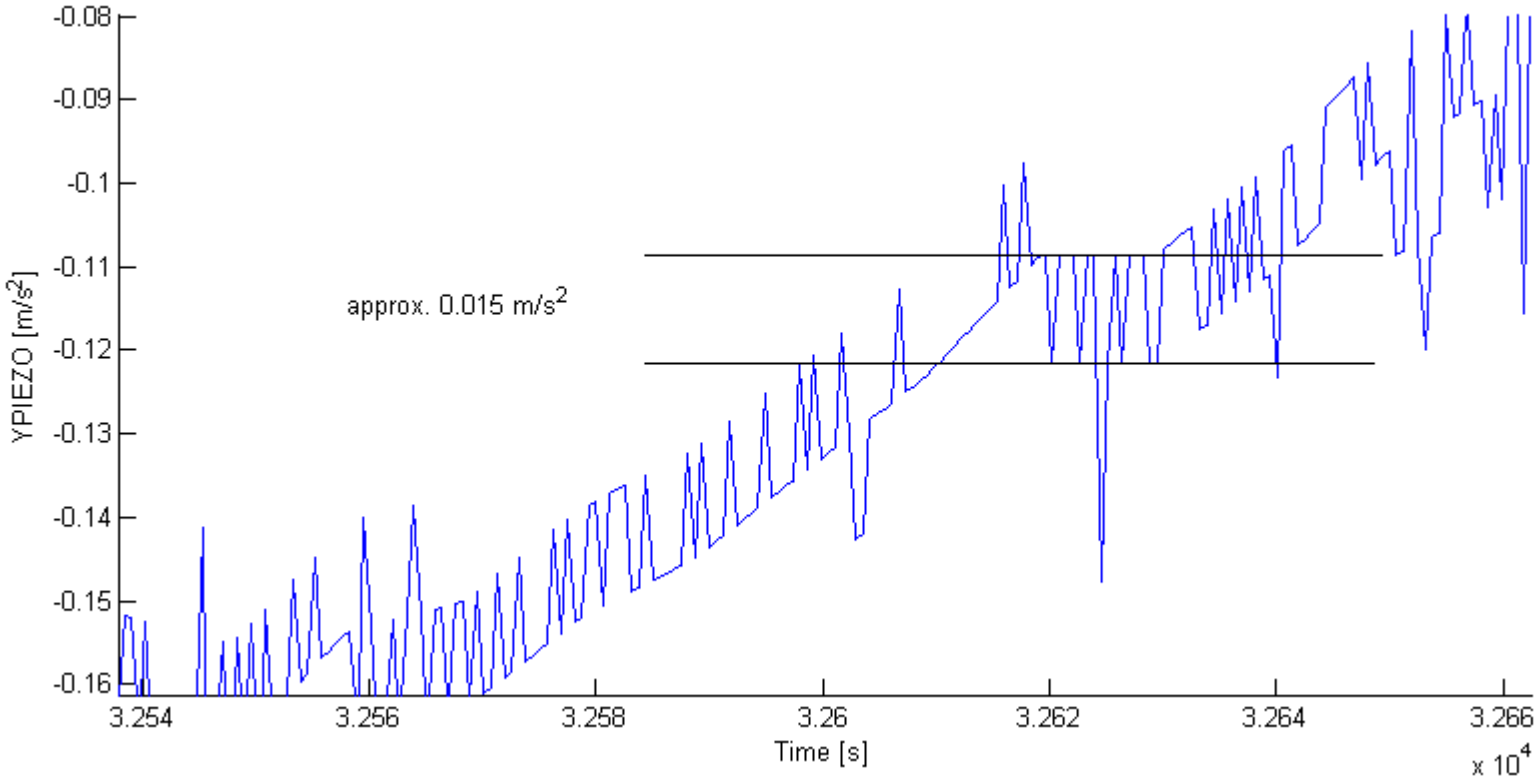
ADC LSB : 0.00488 V

PHYSICAL LSB : 0.00488 / 0.0408163 = **0.23912 m/s^2**

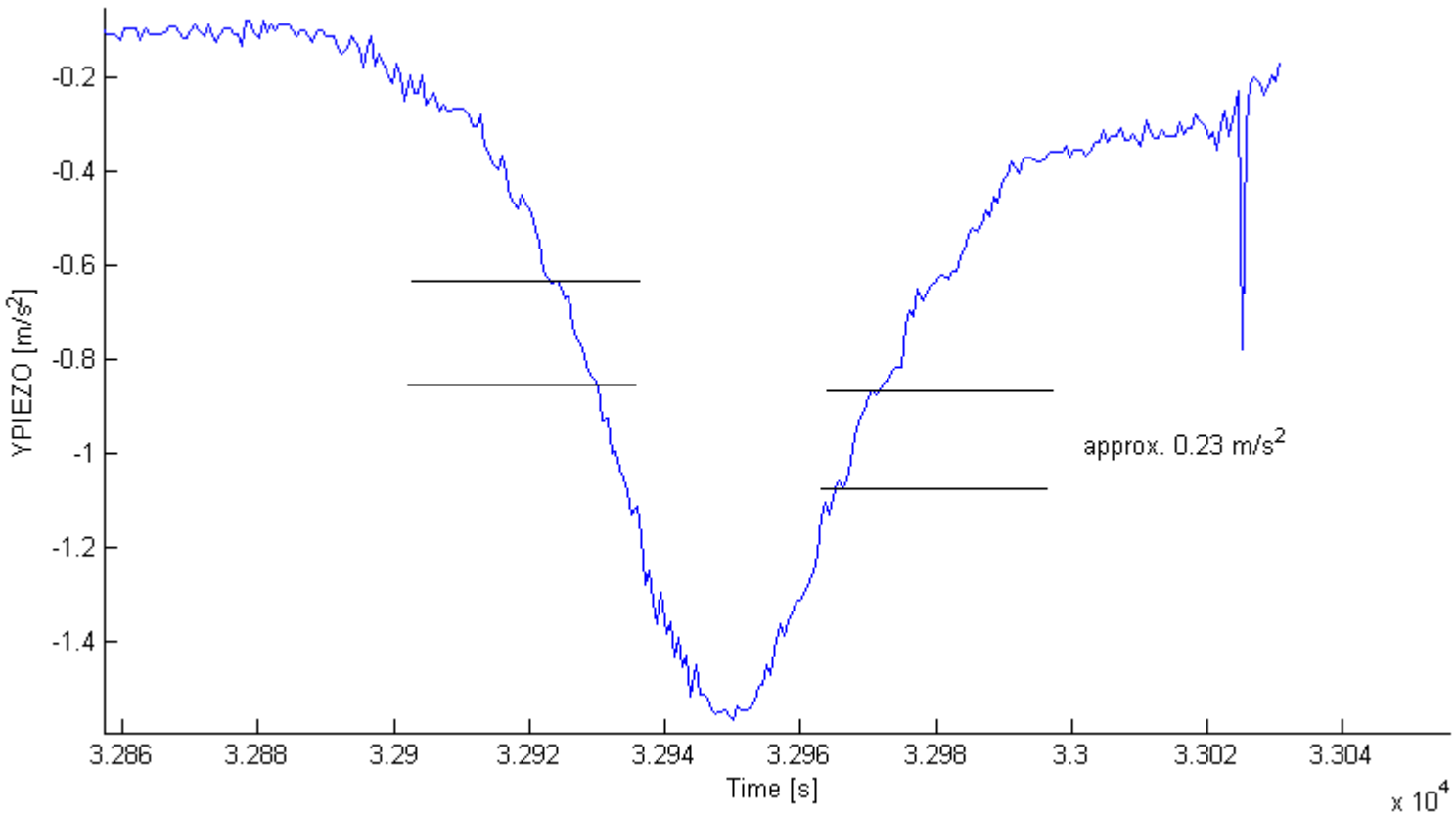
DPU computes a sum over 31 samples and stores it on a 16 BIT WORD so a right shift is needed!

NUMERICAL LSB : 0.23912 * 2 / 31 m/s^2 = **0.01543 m/s^2**

ANNEX 4 : PIEZO data analysis - numerical LSB



ANNEX 4 : PIEZO data analysis - physical LSB



Oscillations on Y/Z PIEZO acc during entry phase corresponds to the physical LSB of the chain smoothed from the 31 samples summation.

Aerodynamic normal forces are too small to be sensed by PIEZOs!
(keep in mind that PIEZO were selected to detect impact)

→ PIEZO are not useful for entry

1. Angle of attack can be estimated from acceleration ratio

$$a_A = a_X \quad , \quad a_N = \sqrt{a_Y^2 + a_Z^2} \;\Rightarrow\; \tan(\alpha) \simeq \frac{a_N}{a_A}$$

2. Acceleration can be integrated twice to compute velocity and position

$$a \;\Rightarrow\; v = \int a\,dt \;\Rightarrow\; p = \int v\,dt$$

3. Using the knowledge of aerodynamics of the probe (AEDB) the atmosphere density can be computed from measured axial acceleration

$$a_X = \frac{C_A \cdot \rho \cdot L \cdot \|v\|^2}{2 \cdot m} \cdot \;\Rightarrow\; \rho = \frac{2 \cdot m \cdot a_X}{C_A \cdot L \cdot \|v\|^2}$$

Steps from 1 to 3 can be iterated many times until convergence

BUT

- Requires al least three accelerometers.
- Coning must be compensated in some manner.
- There's no proof of convergence nor some optimality criterion.

Kalman filter uses the dynamic model to track the probe attitude more accurately!