9th International Planetary Probe Workshop, Toulouse, France, 16th-22nd June 2012

COUPLING OF BLOWING AND ROUGHNESS EFFECTS IN THE SPALART-ALLMARAS TURBULENCE MODEL

Maria Gritsevich[1,2], Daria Kuznetsova[1,2], and Ilias Sibgatullin[2]

[1] Faculty of Mathematics and Mechanics of Lomonosov Moscow State University,

[2] Institute of Mechanics of Lomonosov Moscow State University

INTRODUCTION

During a high speed atmospheric re-entry Thermal Protection System can ablate and develop roughness. A review of surface roughness and blowing influence on aerothermodynamics reveal these effects to be significant, and thus it is important to set up an extended model taking into account both of them. The model for a flow in a turbulent boundary layer on a rough surface was constructed earlier based on the extension of the Spalart–Allmaras turbulence model (Spalart, Aupoix). One of the main suggestions of the Spalart–Allmaras turbulence model is the logarithmic velocity profile in the boundary layer, which is shifted for a rough surface depending on the roughness height. In this study the possibility of taking into account both roughness and blowing in the Spalart-Allmaras model is analyzed. The approach is based on the modification of the variables and boundary conditions of the base Spalart-Allmaras model in the way similar to description of the roughness effects in the Spalart-Allmaras-Aupoix approach. Two ways of including the effect of roughness are considered which use two different velocity laws. The first one uses the formula of Ilegbusi for the velocity which describes the both effects of blowing and roughness. The second one uses the bilogarithmic law for the velocity which holds in case of blowing.

DESCRIPTION OF THE MODEL

BASIC SPALART–ALLMARAS TURBULENCE MODEL

Steady 1-dimensional case: $0 = c_{b1}\tilde{S}^+\tilde{\nu}^+ - c_{w1} f_w\left(\frac{\tilde{\nu}^+}{d^+}\right)^2 + \frac{1}{\sigma}\left[\frac{\partial}{\partial y^+}\left(\tilde{\nu}^+\frac{\partial \tilde{\nu}^+}{\partial y^+}\right) + c_{b2}\left(\frac{\partial \tilde{\nu}^+}{\partial y^+}\right)^2\right]$

taking into account that $(1+\nu_t^+)\frac{\partial u^+}{\partial y^+} = 1$

Definitions: κ – the Karman constant c_w, c_{b1}, c_{b2} – constants of the Spalart–Allmaras model

$\nu_t = f_{\nu_1}\tilde{\nu},\ f_{\nu_1} = \frac{\chi^3}{\chi^3 + c_{\nu_1}^3},\ \chi = \frac{\tilde{\nu}}{\nu}$ $\tilde{S} = \frac{\partial u}{\partial y} + \frac{\tilde{\nu}}{\kappa^2 d^2} f_{\nu 2},\ f_{\nu 2} = 1 - \frac{\chi}{1+\chi f_{\nu_1}}$ d – distance to the nearest wall

In the limit region (far from the wall): $\tilde{\nu} = u_\tau \kappa d,\ \tilde{S} = \frac{u_\tau}{\kappa d}$

Boundary conditions:

$\tilde{\nu} = \tilde{\nu}_w^+$ at the wall

$\tilde{\nu}^+ = \nu_t^+ = \kappa y^+ - 1$ in the limit region

THE EXTENSION FOR ROUGHNESS (SPALART-AUPOIX)

Roughness is supposed to produce a shift Δu^+ as compared to the variables on a smooth surface. This shift can be expressed in two ways:

1) Δu^+= {logarithmic law for velocity on a smooth surface} – {expression for velocity obtained from the numerical solution}

2) Analytical expression from Nikuradse's law

$$\Delta u^+ = \frac{1}{\kappa}\ln h_s^+ + C - B, \kappa = 0.40, C = 5.5$$

$$1 < h_s^+ < 3.5 \Rightarrow B = 5.5 + \frac{1}{\kappa}\ln h_s^+$$
$$3.5 < h_s^+ < 7 \Rightarrow B = 6.59 + 1.52\ln h_s^+$$
$$7 < h_s^+ < 14 \Rightarrow B = 9.58$$
$$14 < h_s^+ < 68 \Rightarrow B = 11.5 - 0.7\ln h_s^+$$
$$68 < h_s^+ \Rightarrow B = 8.48$$

Thus, the dependence is defined: $\Delta u^+(h_s^+) = \phi(\tilde{\nu}_w^+)$

Then: $\boxed{\tilde{\nu}_w^+ = \phi^{-1}(\Delta u^+(h_s^+))}$

1. BILOGARITMIC LAW FOR THE VELOCITY IN CASE OF BLOWING

1. Fully turbulent part of the boundary layer $\tau_{visc} << \tau_{turb}$
2. Prandtl's formula $\tau_{turb} = \rho l^2 \frac{\partial u}{\partial y}, l = \kappa y$
3. Absence of the pressure gradient $\frac{dp}{dx} = 0$

$$\rho\left(u\cancel{\frac{\partial u}{\partial x}} + v\frac{\partial u}{\partial y}\right) = \frac{\partial \tau_{turb}}{\partial y} \Rightarrow v_w u = \frac{\tau_{turb} - \tau_w}{\rho} \Rightarrow \frac{2u_*}{v_w}\left(\left(\frac{v_w u}{u_*^2}+1\right)^{1/2} - 1\right) = \frac{1}{\kappa}\ln\frac{y u_*}{\nu} + C - \frac{2u_*}{v_w}$$

$u = u(y)$, 0, $v = v_w$, C_0

$$u_* = \sqrt{\frac{\tau_w}{\rho}}$$

C_0 - the constant from the logarithmic law

$$\boxed{u = \frac{v_w}{4\kappa^2}\left(\ln\frac{y u_*}{\nu} + \kappa C_0\right)^2 + \frac{u_*}{\kappa}\left(\ln\frac{y u_*}{\nu} + \kappa C_0\right)}$$

The limit case: $\tilde{\nu} = u_\tau \kappa y$ $\tilde{S} = \frac{u_\tau}{\kappa y}$

Variables: $\nu_t = \frac{v_w y}{2}(\ln y^+ + C_0\kappa) + f_{\nu_1}\tilde{\nu}, \tilde{S} = \frac{\partial u}{\partial y} + \frac{\tilde{\nu}}{(\kappa y)^2} f_{\nu_2} - v_w u = \frac{\partial u}{\partial y} + \frac{\nu_\tau}{\kappa^2 y^2} - \frac{v_w}{2\kappa^2 y^2}(\ln y^+ + C_0\kappa)$

Equations:

$$0 = c_{b1}\tilde{S}^+\tilde{\nu}^+ - c_{w1} f_w\left(\frac{\tilde{\nu}^+}{d^+}\right)^2 + \frac{1}{\sigma}\left[\frac{\partial}{\partial y^+}\left(\tilde{\nu}^+\frac{\partial \tilde{\nu}^+}{\partial y^+}\right) + c_{b2}\left(\frac{\partial \tilde{\nu}^+}{\partial y^+}\right)^2\right]$$

$$(1+\nu_t^+)\frac{\partial u^+}{\partial y^+} = v_w^+ u^+ + 1$$

Boundary conditions:

$\tilde{\nu} = \tilde{\nu}_w^+$ at the wall

$\tilde{\nu}^+ = \nu_t^+ - \frac{1}{2}v_w^+ y^+(\ln y^+ + C_0\kappa)$ in the limit region

From the Spalart–Allmaras-Aupoix model: $\Delta u(h_s, v_w) = \phi(\tilde{\nu}_w) \Rightarrow \boxed{\tilde{\nu}_w = \phi^{-1}(\Delta u(h_s, v_w))}$

2. THE VELOCITY LAW OF ILEGBUSI

$$\boxed{\frac{u}{u_\tau} = \frac{1}{\kappa}\ln\frac{Ey\sqrt{\bar{\tau}\rho}}{\mu}}$$

$$\bar{\tau} = \tau_w + \frac{\dot{m}u_\infty^2\kappa}{u_\infty + u_\tau}$$
$$E = \exp(A_R)/h_s^+, A_R = A_R(h_s^+)$$

u_∞ – velocity in the outer layer

$\dot{m}$ – mass transfer from the surface

$\Rightarrow$ (taking into account the law of Nikuradse) $u^+ = \frac{1}{\kappa}\left(\ln y^+ - \ln h_s^+ + \kappa B + \ln\left(\sqrt{1+\frac{\dot{m}^+ u_\infty^{+2}}{1+u_\infty^+}}\right)\right)$

$$\dot{m}^+ = \frac{\dot{m}}{\rho u_\tau} \qquad u_\infty^+ = \frac{u_\infty}{u_\tau}$$

From the Spalart–Allmaras model: $\boxed{\tilde{\nu}_w = \phi^{-1}(\Delta u(h_s, \dot{m}))}$

REFERENCES

Aupoix B., Spalart P.R. Extensions of the Spalart-Allmaras turbulence model to account for wall roughness. Int. J.of Heat and Fluid Flows, 2003, V. 24, pp.454–462.

Baker R.J., Launder B.E. The turbulent boundary layer with foreign gas injection – I. Measurements in zero pressure gradient. Int. J. Heat Mass Transfer, 1974, V. 17, pp.275-291.

Black T.J., Sarnecki A.J. The turbulent boundary layer with suction or injection. Aero. Res. Counc., Lond., Rep. no. 20, 501, 1958.

Ilegbusi J.O. Proposal for wall function for friction and heat transfer in the presence of roughness and mass transfer. Comm. Heat Mass Transfer, 1984, V. 11, pp.569-581.

Kornilov V.I., Boiko A.V. Efficiency of Air Microblowing Through Microperforated Wall. AIAA Journal, 2012, V. 50, No.3, pp.724-732.

Stevenson T.N. A law of the wall for turbulent boundary layers with suction or injection. Coll. Aero. Cranfield, Rep. Aero no. 166, 1963.

Verollet E., Fulachier L. et Dekeyser I. Etude phenomenologique d'une couche limite turbulente avec aspiration et chauffage à la paroi. Int. J. Heat Mass Transfer, 1977, V. 20, pp.107-112.

Voisinet R.L. Influence of roughness and blowing on compressible turbulent boundary layer flow. Rpt. NSWC-TR-79-153, June 1979.

ACKNOWLEDGEMENTS

The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement nº 241992.